A Technology-Gap Model of ‘Premature’ Deindustrialization

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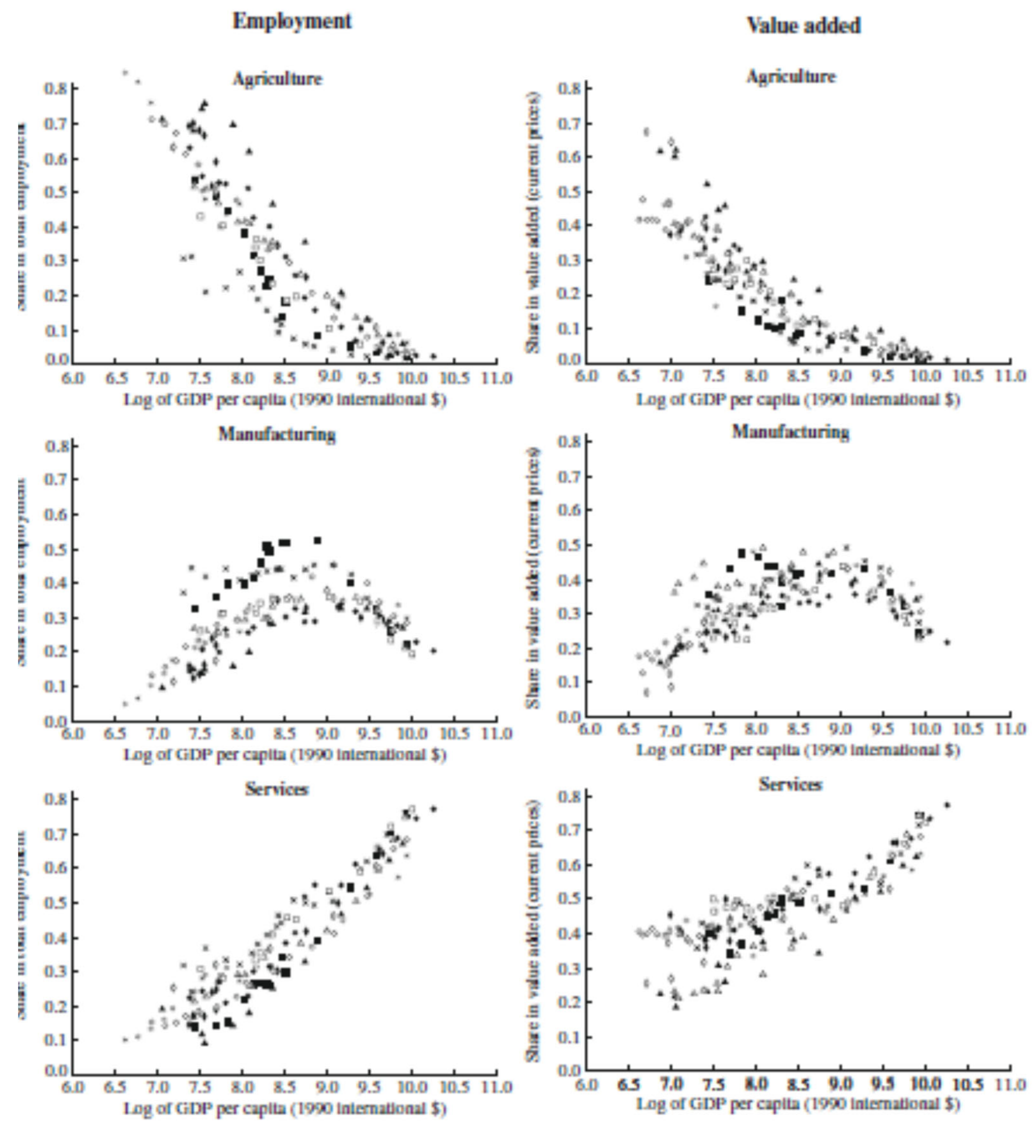
Introduction

Structural Change

As per capita income rises, the employment or value-added shares

- *Fall* in Agriculture
- *Rise* in Services
- *Rise and Fall* in Manufacturing



From Herrendorf-Rogerson-Valentinyi (2014)

Evidence from Long Time Series for the Currently Rich Countries (Belgium, Finland, France, Japan, Korea, Netherlands, Spain, Sweden, United Kingdom, and United States) 1800-2000

Premature Deindustrialization (PD): Rodrik (JEG 2016)

Late industrializers reach their M-peak and start deindustrializing

- *Later* in time
- *Earlier* in per capita income
- with the *lower* peak M-sector shares,

compared to early industrializers.

Rodrik (2016) focuses on documenting the patterns, instead of offering a causal explanation or making normative statements. But

- He speculates that globalization may be a cause.
- He cautions against drawing policy implications, but the word, “premature,” may seem to suggest some types of inefficiency.

In our proposed mechanism,

- PD occurs in the *efficient* equilibrium of a *closed* economy.
- PD is robust to opening up for trade but weakened.

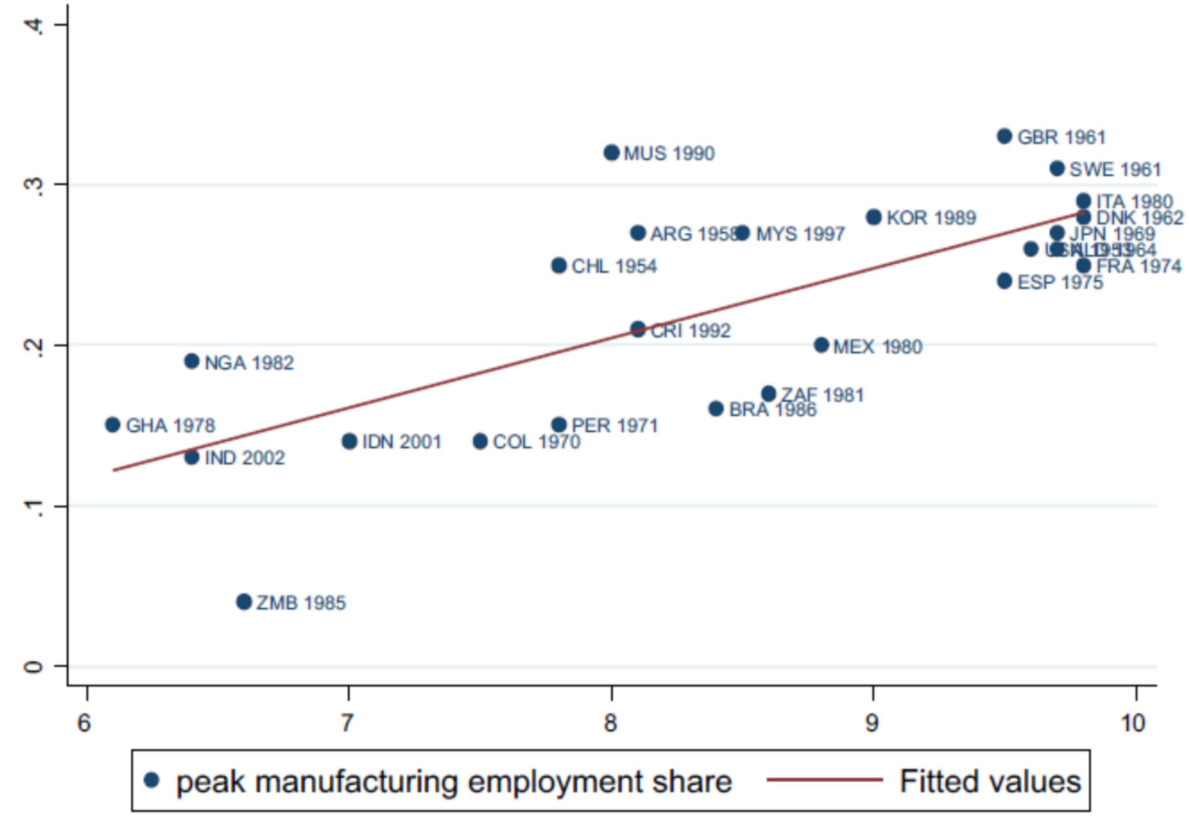


Fig. 5 Income at which manufacturing employment peaks (logs)

This Paper: A Parsimonious Mechanism of Premature Deindustrialization (PD)

In the baseline model,
3 Goods/Sectors: 1=(**A**)griculture, 2=(**M**)anufacturing, 3=(**S**)ervices, *homothetic CES with gross complements* ($\sigma < 1$)

Frontier Technology: $\bar{A}_j(t) = \bar{A}_j(0)e^{g_j t}$, with $g_1 > g_2 > g_3 > 0$ ⇒ a decline of A, a rise of S, and a hump-shaped of M in each country through **the Baumol (relative price) effect**, as in Ngai-Pissarides (2007)

Countries differ in Actual Technology Used: $A_j(t) = \bar{A}_j(t - \lambda_j)$ due to **Adoption Lags,** $(\lambda_1, \lambda_2, \lambda_3)$.

$$A_j(t) = \bar{A}_j(t - \lambda_j) = \bar{A}_j(0)e^{-g_j\lambda_j}e^{g_j t} \implies \frac{\partial}{\partial \lambda_j}\ln\left(A_j(t)\right) = -g_j < 0$$

λ_j has **no "growth" effect, but negative "level" effects,** $e^{-\lambda_j g_j}$, amplified by g_j.

Log-submodularity: g_j magnifies the (negative) impact of the adoption lag on productivity: $\frac{\partial}{\partial g_j}\left(\frac{\partial}{\partial \lambda_j}\ln e^{-g_j\lambda_j}\right) < 0$

We focus on 1-dimension of cross-country heterogeneity: $(\lambda_1, \lambda_2, \lambda_3) = (\theta_1, \theta_2, \theta_3)\lambda$,

- $\lambda \geq 0$, **Technology Gap,** country-specific, as in Krugman (1985); their ability to adopt the frontier technologies.
- $\theta_j > 0$: sector-specific, unlike Krugman (1985); how much λ affects the adoption lag and productivity in each sector.

$$A_j(t) = \bar{A}_j(0)e^{-g_j\theta_j\lambda}e^{g_j t} \implies \frac{\partial}{\partial \lambda}\ln\left(\frac{A_j(t)}{A_k(t)}\right) = -(\theta_j g_j - \theta_k g_k).$$

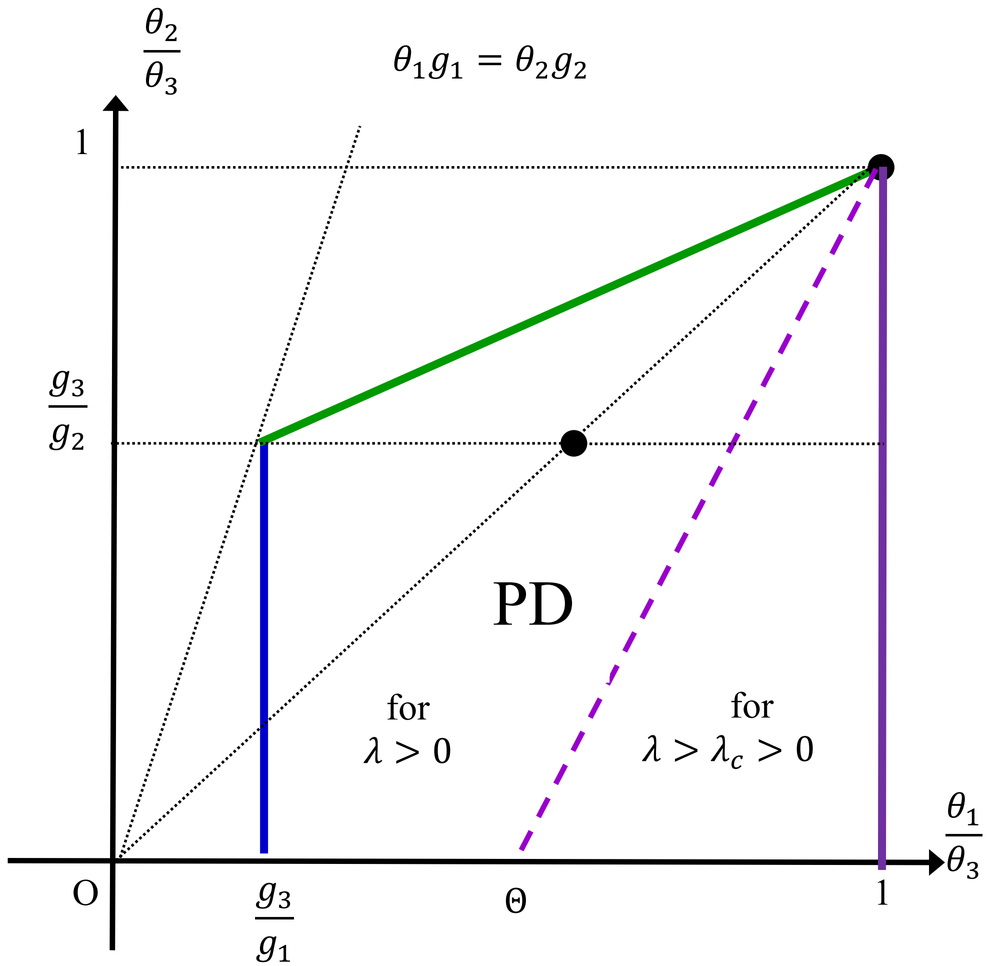
Main Results: Conditions for PD, defined as "A high-λ country reaches its peak later in time, with lower peak M-share at lower per capita income at its M-peak time."

i) $\theta_1 g_1 > \theta_3 g_3$: cross-country productivity difference larger in A than in S.
High relative price of A/low relative price of S in a high-λ country causes a delay.

ii) $\left(1-\frac{g_3}{g_1}\right)\left(\frac{\theta_2}{\theta_3}-1\right)+\left(1-\frac{g_3}{g_2}\right)\left(1-\frac{\theta_1}{\theta_3}\right)<0$:
Technology adoption takes not too long in M.
Not too high relative price of M in a high-λ country keeps the M-share low.

Under the above conditions,

iii) $\theta_1 < \theta_3$: Technology adoption takes longer in S than in A.
Longer adoption lag in S in a high-λ country causes "premature" deindustrialization.



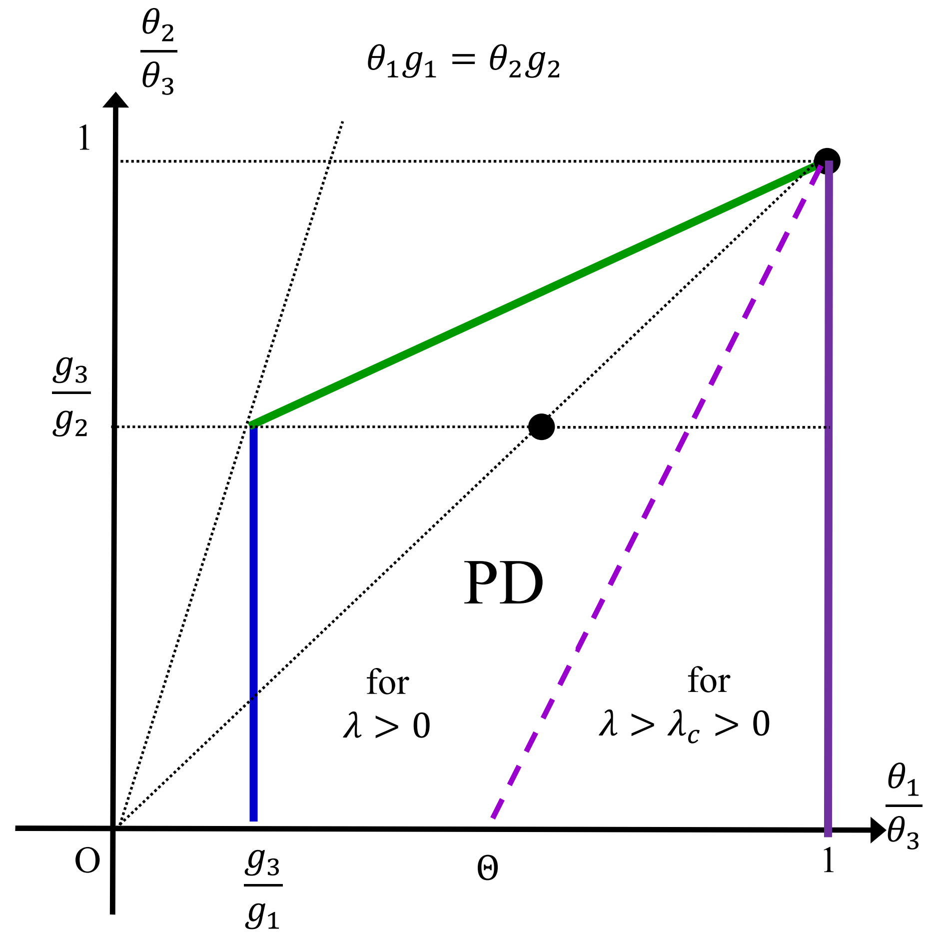
Notes:

✓ No PD if $\theta_1 = \theta_2 = \theta_3$. Latecomers would follow the same path with a delay.

✓ Conditions i) & ii) $\Rightarrow \theta_1 g_1 > \max\{\theta_2 g_2, \theta_3 g_3\}$.
Cross-country productivity difference is the largest in A.
$\theta_2 g_2 - \theta_3 g_3$ can be either positive or negative.
negative when calibrated to match Rodrik's (2016, Table10) findings.

In sum, PD occurs because a high-λ country is:

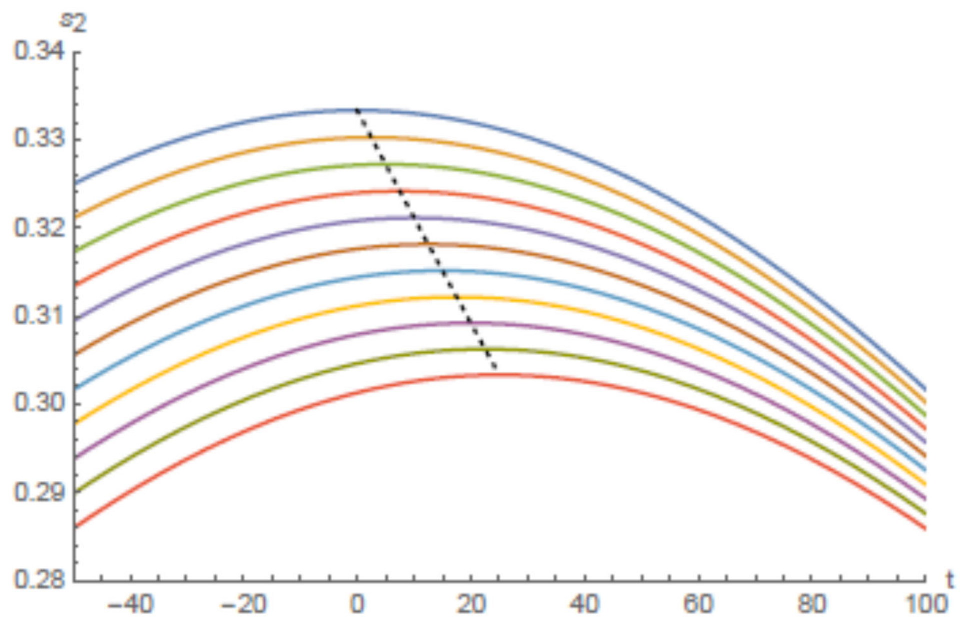
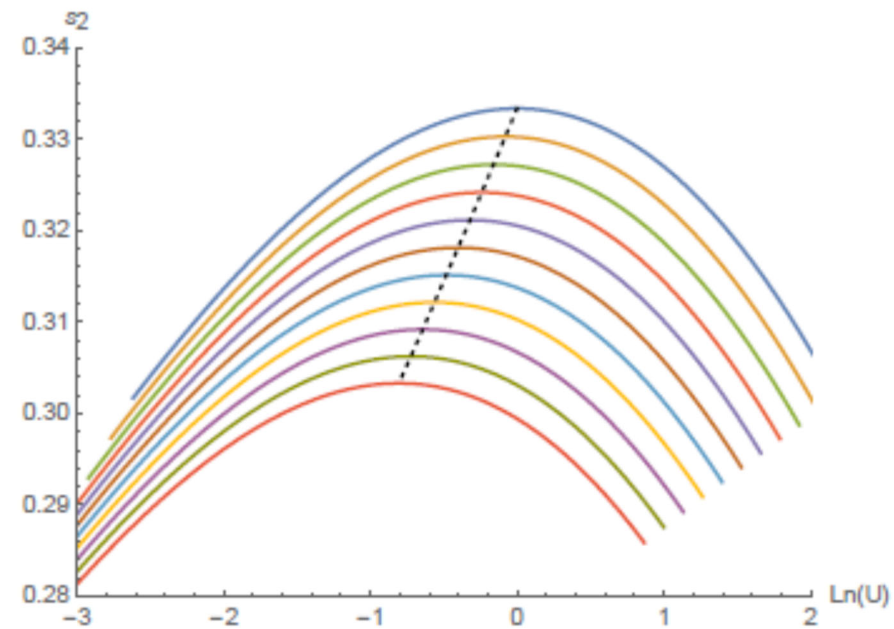
i) The worse in A than in S, which causes it to peak later in time.
ii) Its adoption takes the longest in S, which causes it to deindustrialize prematurely.
iii) Doing ok in M, which explains why its peak M-share is lower.



PD should not be viewed as the prima-facie evidence that the M-sectors in developing countries are doing badly.
On the contrary, they are doing fine, relatively to their A and S-sectors, according to this model.

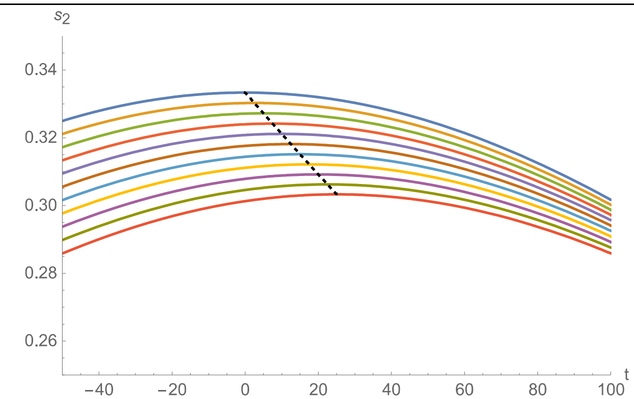
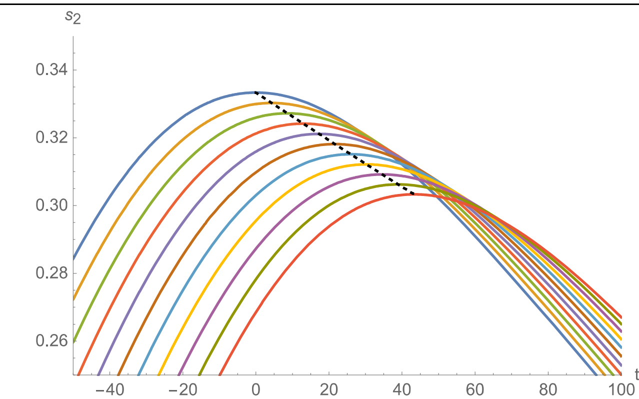
A Numerical Illustration.

$\theta_1 = \theta_2 < \theta_3 = 1$ with $g_1 = 3.6\% > g_2 = 2.4\% > g_3 = 1.2\%$; $\sigma = 0.6$; Labor share $= 2/3$. We set the other parameters, w.l.o.g., so that the peak time, $\hat{t} = 0$ and the peak time income per capita, $U(\hat{t}) = 1$ if $\lambda = 0$.

Example 2a	$(t, s_2(t))$	$(\ln U(t), s_2(t))$
$\frac{\theta_1}{\theta_3} = \frac{\theta_2}{\theta_3} = 0.5 = \frac{g_3}{g_2}$ $\Longrightarrow \theta_1 g_1 > \theta_2 g_2 = \theta_3 g_3$		
— λ=0 — λ=10 — λ=20 — λ=30 — λ=40 — λ=50 — λ=60 — λ=70 — λ=80 — λ=90 — λ=100		

1st Extension: Adding **the Engel Effect** with **Nonhomothetic CES** (a la Comin-Lashkari-Mestieri)

Nonhomotheticity changes the shape of trajectories greatly, but not on how technology gaps, λ, affects the peak values.

<table>
<tr><th></th><th>Homothetic ($\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$)</th><th>Nonhomothetic ($\varepsilon_1 = .4 < \varepsilon_2 = 1 < \varepsilon_3 = 1.6$)</th></tr>
<tr><td>$(t, s_2(t))$</td><td>
</td><td>
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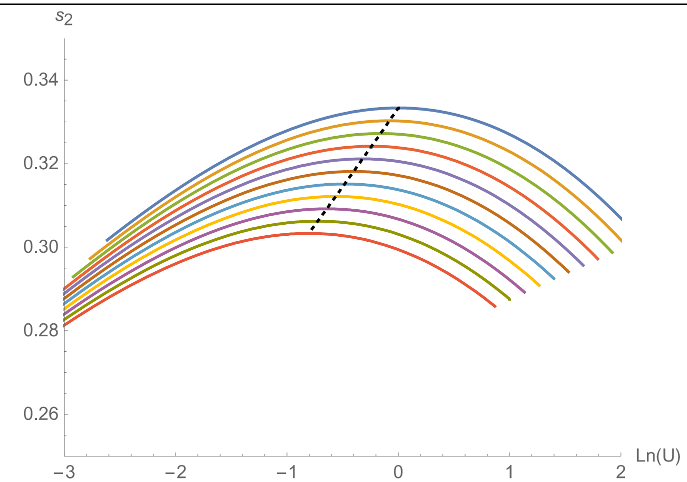
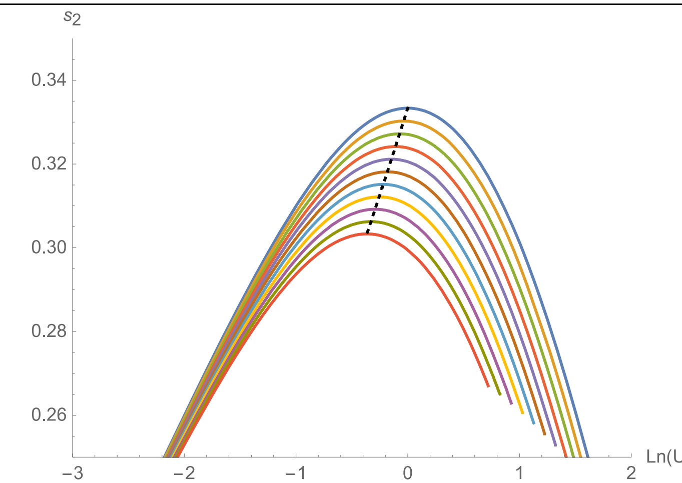
<tr><td>$(\ln U(t), s_2(t))$</td><td>
</td><td>
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We also show that the Engel effect *alone* could not generate PD *without counterfactual implications*.

2nd Extension: International Trade

One implication of our mechanism for PD (consistent with the empirical evidence):

$$\frac{\partial}{\partial \lambda} \ln\left(\frac{A_1(t)}{A_2(t)}\right) = -(\theta_1 g_1 - \theta_2 g_2) < 0.$$

- A low-λ country has comparative advantage in A and a high-λ country has comparative advantage in M.
- Opening up for trade allows a high-λ country to export M to a low-λ country.
- Our mechanism for PD is weakened by opening up for trade, but PD continues to hold, as long as the trade cost is not too small.
- Consistent with the findings that East Asia "suffers" less from PD (Rodrik 2016).

Under our mechanism, PD occurs *not because of*, but *in spite of* international trade.

3rd Extension: Introducing Catching-up

$$A_j(t) = \bar{A}_j(0)e^{g_j(t-\theta_j\lambda_t)}, \quad \text{where} \quad \lambda_t = \lambda_0 e^{-g_\lambda t},$$

Countries differ only in the *initial* value, $\boldsymbol{\lambda_0}$, converging exponentially over time at **the same rate,** $g_\lambda > 0$

<table>
<tr><th>Peak Time</th><th>Peak M-Share</th><th>Peak time Per Capita Income</th></tr>
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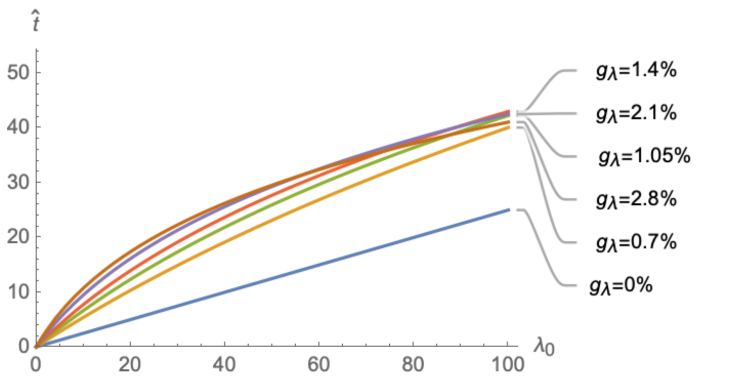

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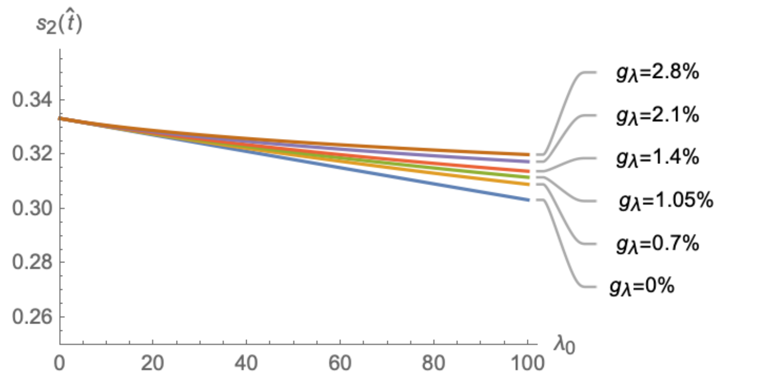

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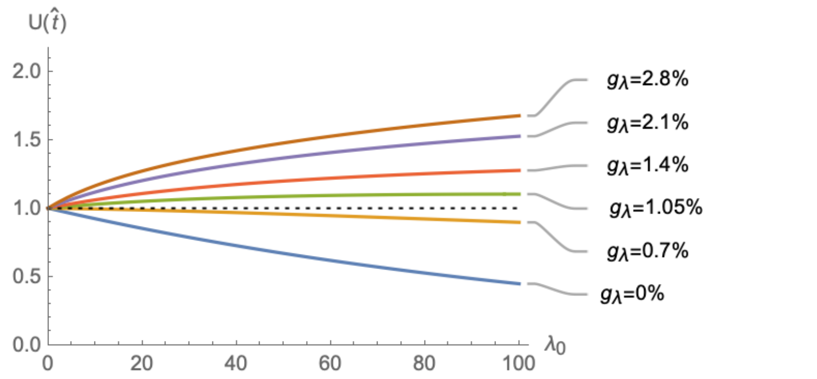

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Higher-λ countries

- peak later in time,
- have lower peak M-shares
- have lower peak time per capita income, unless g_λ is too large.

(**Very Selective) Literature Review.** Herrendorf-Rogerson-Valentinyi (14) for a survey on structural change.

Related to The Baseline Model

Premature Deindustrialization, Dasgupta-Singh (06), Palma (14), **Rodrik (16)**
The Baumol Effect: Baumol (67), **Ngai-Pissarides (07),** Nordhaus (08)
Cross-country heterogeneity in technology development

- *Distance to the frontier:* **Krugman** (85), Acemolgu-Aghion-Zilibotti (06)
- *Log-supermodularity:* **Krugman** (85), Matsuyama (05), Costinot (09), Costinot-Vogel (15)
- *Productivity difference across countries the largest in A:* Caselli (05), Gollin et.al. (14, AERP&P)
- *Small adoption lags in M;* Rodrik (2013)

Related to Three Extensions

The Engel Effect (Nonhomotheticity); Murphy et.al. (89), Matsuyama (92,02), Kongsamut et.al. (01), Foellmi-Zweimueller (08), Buera-Kaboski (09,12), Boppart (14), **Comin-Lashkari-Mestieri (21),** Matsuyama (19), Lewis et.al. (21), Bohr-Mestieri-Yavuz (21)
Open Economy Implications: Matsuyama (92,09), Uy-Yi-Zhang (13), Sposi-Yi-Zhang (19), Fujiwara-Matsuyama (WinP)
Catching-Up/Technology Diffusion: Acemoglu (08), Comin-Mestieri (18)

The Issues We Abstract From

Sector-level productivity growth rate differences across countries: Huneeus-Rogerson (20)
Endogenous growth, externalities, Matsuyama (92).
Sectoral wedges/misallocation: Caselli (05), Gollin et.al. (14 QJE) and many others
Nominal vs. Real expenditure; Employment vs. Value Added shares; Compatibility with aggregate balance growth, investment vs consumption, sector-specific factor intensities, skill premium, home production, productivity slowdown, etc.

Structural Change, the Baumol Effect, and Adoption Lags

Three Complementary Goods/Competitive Sectors, $j = 1, 2, 3$

Sector-1 = **(A)**griculture, Sector-2 = **(M)**anufacturing, Sector-3 = **(S)**ervices.

Demand System: L Identical HH, each endowed with 1 unit of mobile labor, earning the wage w & κ_j units of managerial skills, specific to j, each earning the rent, ρ_j.

Budget Constraint:

$$\sum_{j=1}^{3} p_j c_j \leq E \equiv w + \sum_{j=1}^{3} \rho_j \kappa_j = \frac{1}{L}\sum_{j=1}^{3} p_j Y_j$$

CES Preferences:

$$U(c_1, c_2, c_3) = \left[\sum_{j=1}^{3} (\beta_j)^{\frac{1}{\sigma}} (c_j)^{1-\frac{1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

with $\beta_j > 0$ and $0 < \sigma < 1$ (gross complementarity)

Expenditure Shares:

$$m_j \equiv \frac{p_j c_j}{E} = \beta_j \left(\frac{p_j}{P}\right)^{1-\sigma}; \quad P = \left[\sum_{k=1}^{3} \beta_k (p_k)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

Real Per Capita Income

$$U = \frac{E}{P} = \left[\sum_{k=1}^{3} \beta_k \left(\frac{E}{p_k}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}.$$

Three Competitive Sectors: Production

Cobb-Douglas

$$Y_j = \tilde{A}_j(\kappa_j L)^{\alpha}(L_j)^{1-\alpha} = A_j(L)^{\alpha}(L_j)^{1-\alpha} = LA_j(s_j)^{1-\alpha}$$

where $A_j \equiv \tilde{A}_j(\kappa_j)^{\alpha}$. $\alpha \in [0,1)$: the span of control parameter, which introduces diminishing returns in labor.

Labor Share $\dfrac{wL_j}{p_j Y_j} = 1 - \alpha$ **Profit (Managerial Rent) Share** $\dfrac{\rho_j \kappa_j L}{p_j Y_j} = \alpha$

Sectoral Share in Employment

$$s_j \equiv \frac{L_j}{L}; \qquad \sum_{j=1}^{3} s_j = 1$$

Sectoral Sector in Value-Added

$$\frac{p_j Y_j}{EL} = \frac{p_j Y_j}{\sum_{k=1}^{3} p_k Y_k},$$

$$\Longrightarrow \frac{p_j Y_j}{EL} = s_j = \left(\frac{p_j A_j}{E}\right)^{1/\alpha}; \;\; E = \left[\sum_{k=1}^{3} (p_k A_k)^{\frac{1}{\alpha}}\right]^{\alpha}.$$

Note: The value-added shares are equal to the employment shares.

Equilibrium: The expenditure shares are equal to the employment shares (=value-added shares).

$$\beta_j \left(\frac{p_j}{P}\right)^{1-\sigma} = m_j = \frac{p_j Y_j}{EL} = s_j = \left(\frac{p_j A_j}{E}\right)^{1/\alpha}$$

which lead to

Equilibrium Shares

$$s_j = \frac{\left[\beta_j^{\frac{1}{\sigma-1}} A_j\right]^{-a}}{\sum_{k=1}^{3}\left[\beta_k^{\frac{1}{\sigma-1}} A_j\right]^{-a}}$$

Per Capita Income

$$U = \left\{\sum_{k=1}^{3}\left[\beta_k^{\frac{1}{\sigma-1}} A_j\right]^{-a}\right\}^{-\frac{1}{a}}$$

where

$$a \equiv \frac{1-\sigma}{1-\alpha(1-\sigma)} = -\frac{\partial \log(s_j/s_k)}{\partial \log(A_j/A_k)} > 0,$$

which captures how much relatively *high* productivity in a sector contributes to its relatively *low* equilibrium share. α magnifies this effect by increasing a.

Productivity Growth:

$$A_j(t) = \bar{A}_j(t - \lambda_j) = \bar{A}_j(0)e^{g_j(t-\lambda_j)} = \bar{A}_j(0)e^{-\lambda_j g_j}e^{g_j t}$$

$\bar{A}_j(t) = \bar{A}_j(0)e^{g_j t}$: **Frontier Technology** in j, with a constant **growth rate** $g_j > 0$.

$A_j(t) = \bar{A}_j(t - \lambda_j)$; λ_j = **Adoption Lag** in j.

- λ_j has **no "growth" effect,** but has **a negative "level" effect,** $e^{-\lambda_j g_j}$, which is proportional to g_j.

Key: Log-submodularity, $\frac{\partial}{\partial g_j}\left(\frac{\partial}{\partial \lambda_j}\ln e^{-\lambda_j g_j}\right) < 0$: g_j magnifies the negative effect of the adoption lag on productivity

✓ A large adoption lag doesn't matter much in a sector with slow productivity growth.

✓ Even a small adoption lag matters a lot in a sector with fast productivity growth.

$$U(t) = \left\{\sum_{k=1}^{3}\left[\beta_k^{\frac{1}{\sigma-1}}A_k(t)\right]^{-a}\right\}^{-\frac{1}{a}} = \left\{\sum_{k=1}^{3}\tilde{\beta}_k e^{-a g_k(t-\lambda_k)}\right\}^{-\frac{1}{a}}, \ where\ \tilde{\beta}_k \equiv \left(\frac{\beta_k^{\frac{1}{1-\sigma}}}{\bar{A}_k(0)}\right)^{a} > 0.$$

Longer adoption lags would shift down the time path of $U(t)$.

$$g_U(t) \equiv \frac{U'(t)}{U(t)} = \sum_{k=1}^{3} g_k s_k(t)$$

The growth rate in per capita income is the weighted average of the sectoral growth rates.

Relative Prices:

$$\left(\frac{p_j(t)}{p_k(t)}\right)^{1-\sigma} = \left[\left(\frac{\beta_j}{\beta_k}\right)^{-\alpha} \frac{\bar{A}_j(0)}{\bar{A}_k(0)}\right]^{-a} e^{a(\lambda_j g_j - \lambda_k g_k)} e^{a(g_k - g_j)t} \Longrightarrow \frac{d \ln\left(\frac{p_j(t)}{p_k(t)}\right)}{dt} = \frac{a(g_k - g_j)}{1-\sigma}$$

Relative Growth Effect: $p_j(t)/p_k(t)$ is de(in)creasing over time if $g_j > (<) g_k$. Speed independent of λ_j and λ_k.
Relative Level Effect: A higher $\lambda_j g_j - \lambda_k g_k$ raises $p_j(t)/p_k(t)$ at any point in time.
Note: For a fixed $\lambda_j > 0$, a higher g_j makes the relative price of j higher (though declining faster).

Relative Shares:

$$\frac{s_j(t)}{s_k(t)} = \frac{\beta_j}{\beta_k}\left(\frac{p_j(t)}{p_k(t)}\right)^{1-\sigma} = \frac{\tilde{\beta}_j}{\tilde{\beta}_k} e^{a(\lambda_j g_j - \lambda_k g_k)} e^{a(g_k - g_j)t} \Longrightarrow \frac{d \ln\left(\frac{s_j(t)}{s_k(t)}\right)}{dt} = a(g_k - g_j)$$

Relative Growth Effect: $s_j(t)/s_k(t)$ is de(in)creasing over time if $g_j > (<) g_k$. Speed independent of λ_j and λ_k.
Shift from faster growing sectors to slower growing sectors over time.
Relative Level Effect: A higher $\lambda_j g_j - \lambda_k g_k$ raises $s_j(t)/s_k(t)$ at any point in time.
Note: For a fixed $\lambda_j > 0$, a higher g_j makes the relative share of j higher (though declining faster).

Structural Change with the Baumol (Relative Price) Effect: Let $g_1 > g_2 > g_3 > 0$

Decline of Agriculture: $s_1(t)$ is decreasing in t, because

$$\frac{1}{s_1(t)} - 1 = \frac{s_2(t)}{s_1(t)} + \frac{s_3(t)}{s_1(t)} = \left[\frac{\tilde{\beta}_2}{\tilde{\beta}_1} e^{a(\lambda_2 g_2 - \lambda_1 g_1)}\right] e^{a(g_1 - g_2)t} + \left[\frac{\tilde{\beta}_3}{\tilde{\beta}_1} e^{a(\lambda_3 g_3 - \lambda_1 g_1)}\right] e^{a(g_1 - g_3)t}$$

Rise of Services: $s_3(t)$ is increasing in t, because

$$\frac{1}{s_3(t)} - 1 = \frac{s_1(t)}{s_3(t)} + \frac{s_2(t)}{s_3(t)} = \left[\frac{\tilde{\beta}_1}{\tilde{\beta}_3} e^{a(\lambda_1 g_1 - \lambda_3 g_3)}\right] e^{-a(g_1 - g_3)t} + \left[\frac{\tilde{\beta}_2}{\tilde{\beta}_3} e^{a(\lambda_2 g_2 - \lambda_3 g_3)}\right] e^{-a(g_2 - g_3)t}$$

Rise and Fall of Manufacturing: $s_2(t)$ is hump-shaped in t, because

$$\frac{1}{s_2(t)} - 1 = \frac{s_1(t)}{s_2(t)} + \frac{s_3(t)}{s_2(t)} = \left[\frac{\tilde{\beta}_1}{\tilde{\beta}_2} e^{a(\lambda_1 g_1 - \lambda_2 g_2)}\right] e^{-a(g_1 - g_2)t} + \left[\frac{\tilde{\beta}_3}{\tilde{\beta}_2} e^{a(\lambda_3 g_3 - \lambda_2 g_2)}\right] e^{a(g_2 - g_3)t}.$$

Hump-shaped due to the two opposing forces: $g_1 > g_2$ pushes labor out of A to M; $g_2 > g_3$ pulls labor out of M to S.

$$s_2'(t) \gtreqless 0 \Leftrightarrow (g_1 - g_2)s_1(t) \gtreqless (g_2 - g_3)s_3(t) \Leftrightarrow g_U(t) = \sum_{k=1}^{3} g_k s_k(t) \gtreqless g_2$$

Initially, $\frac{s_1(t)}{s_3(t)}$ is large; the 1st force is stronger. As $\frac{s_1(t)}{s_3(t)}$ declines over time, the 2nd force becomes stronger eventually.

Characterizing Manufacturing Peak: "^" indicates the peak.

$$s_2'(\hat{t}) = 0 \Leftrightarrow (g_1 - g_2)s_1(\hat{t}) = (g_2 - g_3)s_3(\hat{t}) \Leftrightarrow g_U(\hat{t}) = g_2$$

Peak Time: From $(g_1 - g_2)s_1(\hat{t}) = (g_2 - g_3)s_3(\hat{t})$

$$\hat{t} = \frac{\lambda_1 g_1 - \lambda_3 g_3}{g_1 - g_3} + \hat{t}_0, \qquad \text{where } \hat{t}_0 \equiv \frac{1}{a(g_1 - g_3)} \ln\left[\left(\frac{g_1 - g_2}{g_2 - g_3}\right)\frac{\tilde{\beta}_1}{\tilde{\beta}_3}\right]$$

Two Normalizations: Without any loss of generality,

$$\hat{t}_0 = 0 \Leftrightarrow \frac{g_2 - g_3}{g_1 - g_2} = \frac{\tilde{\beta}_1}{\tilde{\beta}_3} \equiv \left[\left(\frac{\beta_1}{\beta_3}\right)^{\frac{1}{1-\sigma}} \frac{\bar{A}_3(0)}{\bar{A}_1(0)}\right]^a$$

The calendar time is reset so that its M-peak would be reached at $\hat{t} = 0$ in the absence of the adoption lags.

$$U(0) = 1 \text{ for } \lambda_1 = \lambda_2 = \lambda_3 = 0 \Leftrightarrow \sum_{k=1}^{3} \tilde{\beta}_k = \sum_{k=1}^{3} \left(\frac{\beta_k^{\frac{1}{1-\sigma}}}{\bar{A}_k(0)}\right)^a = 1.$$

We use the peak time per capita income in the absence of the adoption lags as the *numeraire.*

Note: Under these normalizations, the peak time share of sector-k in the absence of the adoption lags would be $\tilde{\beta}_k$.

Then,

Peak Time

$$\hat{t} = \frac{\lambda_1 g_1 - \lambda_3 g_3}{g_1 - g_3}.$$

Peak M-Share

$$\frac{1}{\widehat{s_2}} = 1 + \left(\frac{\tilde{\beta}_1}{\tilde{\beta}_2}\right) e^{a(g_1-g_2)\left(\frac{\lambda_1 g_1 - \lambda_2 g_2}{g_1 - g_2} - \hat{t}\right)} + \left(\frac{\tilde{\beta}_3}{\tilde{\beta}_2}\right) e^{a(g_2-g_3)\left(\hat{t} - \frac{\lambda_2 g_2 - \lambda_3 g_3}{g_2 - g_3}\right)}$$

Peak Time Per Capita Income

$$\widehat{U} = \left\{\sum_{k=1}^{3} \tilde{\beta}_k e^{-a g_k (\hat{t} - \lambda_k)}\right\}^{-\frac{1}{a}}$$

So far, we've looked at the impacts of adoption lags in each country in isolation.

Obviously, if we allow ourselves 3-dimension of country heterogeneity, $(\lambda_1, \lambda_2, \lambda_3)$, we can perfectly account for $(\hat{t}, \widehat{s_2}, \widehat{U})$ in each country.

Now, we restrict ourselves to 1-dimension of country heterogeneity, "technology gap," which generate cross-country variations in adoption lags, and study the cross-country implications.

Technology Gaps and Premature Deindustrialization

Consider the world with many countries with

$$(\lambda_1, \lambda_2, \lambda_3) = (\theta_1, \theta_2, \theta_3)\lambda$$

$\lambda \geq 0$: **Technology Gap,** Country-specific

$\theta_j > 0$: Sector-specific, capturing the inherent difficulty of technology adoption, common across countries

- **Countries differ only in 1-dimension**, λ, in their ability to adopt the frontier technologies.
- $\theta_j > 0$ determines how much the technology gap affects the adoption lag in that sector.

$$\frac{A_j(t)}{A_k(t)} = \frac{\bar{A}_j(0)}{\bar{A}_k(0)} e^{-(\theta_j g_j - \theta_k g_k)\lambda} e^{(g_j - g_k)t} \Rightarrow \frac{\partial}{\partial \lambda} \ln\left(\frac{A_j(t)}{A_k(t)}\right) = -(\theta_j g_j - \theta_k g_k)$$

Cross-country productivity difference is larger in sector-j than in sector-k if $\theta_j g_j > \theta_k g_k$.

Proposition 1: Peak Values under the Baumol Effect only

Peak Time:

$$\hat{t}(\lambda) = \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3}\lambda.$$

Peak M-Share:

$$\frac{1}{\widehat{s_2}(\lambda)} = 1 + \left(\frac{\tilde{\beta}_1}{\tilde{\beta}_2}\right) e^{a(g_1 - g_2)\left(\frac{\theta_1 g_1 - \theta_2 g_2}{g_1 - g_2}\lambda - \hat{t}(\lambda)\right)} + \left(\frac{\tilde{\beta}_3}{\tilde{\beta}_2}\right) e^{a(g_2 - g_3)\left(\hat{t}(\lambda) - \frac{\theta_2 g_2 - \theta_3 g_3}{g_2 - g_3}\lambda\right)}$$

Peak Time Per Capita Income:

$$\widehat{U}(\lambda) = \left\{\sum_{k=1}^{3} \tilde{\beta}_k e^{-a g_k[\hat{t}(\lambda) - \theta_k \lambda]}\right\}^{-\frac{1}{a}}$$

Proposition 2: Conditions for PD with the Baumol (Relative Price) Effect

$$\hat{t}'(\lambda) > 0 \text{ for all } \lambda > 0 \Leftrightarrow \theta_1 g_1 > \theta_3 g_3.$$

With $\theta_1 g_1 > \theta_3 g_3$, the price of A is relatively higher than the price of S in a high-λ country, which delays the peak.

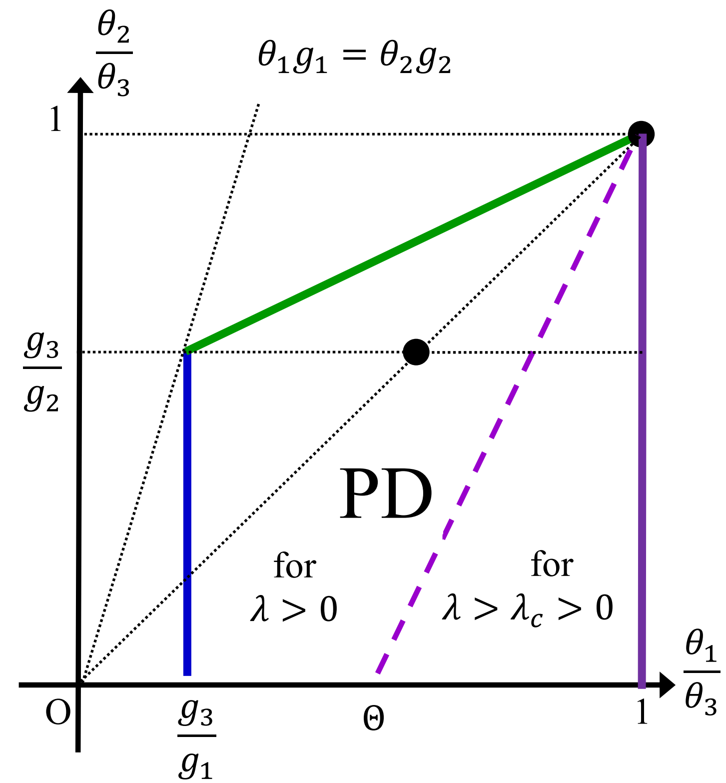
$$\widehat{s_2}'(\lambda) < 0 \text{ for all } \lambda > 0 \Leftrightarrow \left(1 - \frac{g_3}{g_1}\right)\left(\frac{\theta_2}{\theta_3} - 1\right) + \left(1 - \frac{g_3}{g_2}\right)\left(1 - \frac{\theta_1}{\theta_3}\right) < 0$$

With a low θ_2, which has no effect on $\hat{t}$, the price of M is low relative to both A & S in a high-λ country, which keeps the M-share low.

Under the above condition,

$$\widehat{U}'(\lambda) < 0;\ \widehat{U}(\lambda) < \widehat{U}(0) \text{ for } \lambda > \lambda_c \geq 0 \Leftrightarrow \theta_1 < \theta_3 \Leftrightarrow \hat{t}(\lambda) < \theta_1 \lambda < \theta_3 \lambda$$

With $\theta_1 < \theta_3$, the time delay in the peak in a high-λ country is not long enough to make up for the lagging productivity, that is deindustrialization is "premature."



- $\theta_1 g_1 > \max\{\theta_2 g_2, \theta_3 g_3\}$. (productivity differences the largest in A) .
- $\theta_2 g_2 - \theta_3 g_3$ can be either positive or negative.
- $\max\{\theta_1, \theta_2\} < \theta_3$. (adoption lag the longest in S).

Some Examples

Example 1: No Premature Deindustrialization (PD)

Uniform Adoption Lags, as in Krugman (1985)

$$\theta_1 = \theta_2 = \theta_3 = 1 \iff \lambda_1 = \lambda_2 = \lambda_3 = \lambda > 0$$

$$\Longrightarrow \hat{t}(\lambda) = \lambda; \quad \widehat{s_2}(\lambda) = \tilde{\beta}_2; \quad \widehat{U}(\lambda) = 1$$

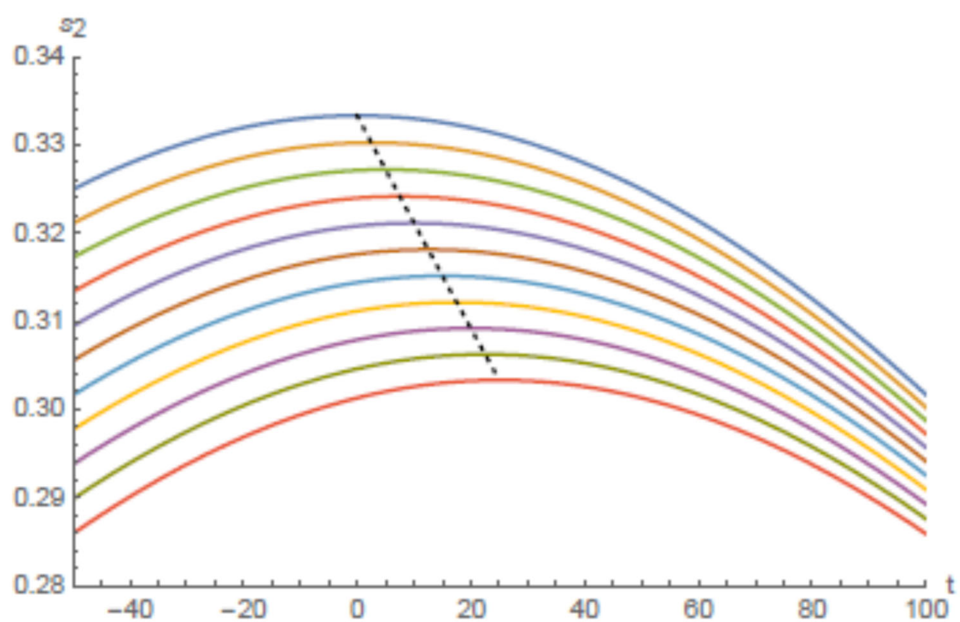
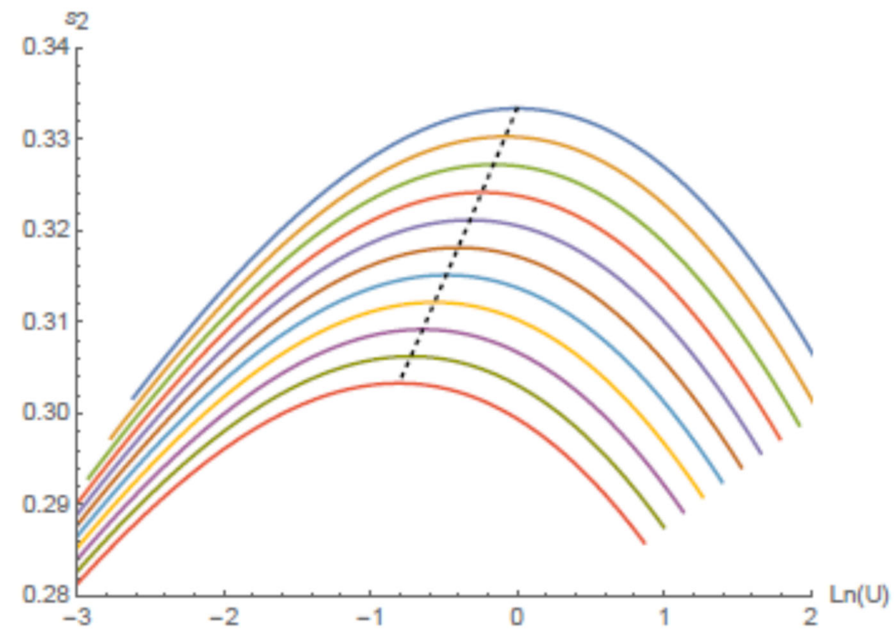
- The country's technology gap causes a delay in the peak time, $\hat{t}$, by $\lambda > 0$.
- The peak M-share & the peak time per capita income unaffected.

Each country follows the same development path of early industrializers *with a delay*. No PD!!

Thus, the technology gap must have differential impacts on the adoption lags across sectors.

A Numerical Illustration.

$\theta_1 = \theta_2 < \theta_3 = 1$ with $g_1 = 3.6\% > g_2 = 2.4\% > g_3 = 1.2\%$; $\sigma = 0.6$; Labor share $= 2/3$. We set the other parameters, w.l.o.g., so that the peak time, $\hat{t} = 0$ and the peak time income per capita, $U(\hat{t}) = 1$ if $\lambda = 0$.

Example 2	$(t, s_2(t))$	$(\ln U(t), s_2(t))$
$\frac{\theta_1}{\theta_3} = \frac{\theta_2}{\theta_3} = 0.5 = \frac{g_3}{g_2}$ $\Longrightarrow \theta_1 g_1 > \theta_2 g_2 = \theta_3 g_3$		
λ=0 λ=10 λ=20 λ=30 λ=40 λ=50 λ=60 λ=70 λ=80 λ=90 λ=100		

Some Calibrations:

Rodrik (2016) divided countries into pre-1990 peaked vs. post-1990 peaked.
From his Fig.5, $\hat{t}(\lambda) = 25$ years. From his Table 10,
For the employment shares, $\widehat{s_2}(0) = 21.5\% > \widehat{s_2}(\lambda) = 18.9\%$; $\widehat{U}(\lambda)/\widehat{U}(0) = \widehat{U}(\lambda) = 4273/11048$.
For the value-added shares, $\widehat{s_2}(0) = 27.9\% > \widehat{s_2}(\lambda) = 24.1\%$. $\widehat{U}(\lambda)/\widehat{U}(0) = \widehat{U}(\lambda) = 20537/47099$.

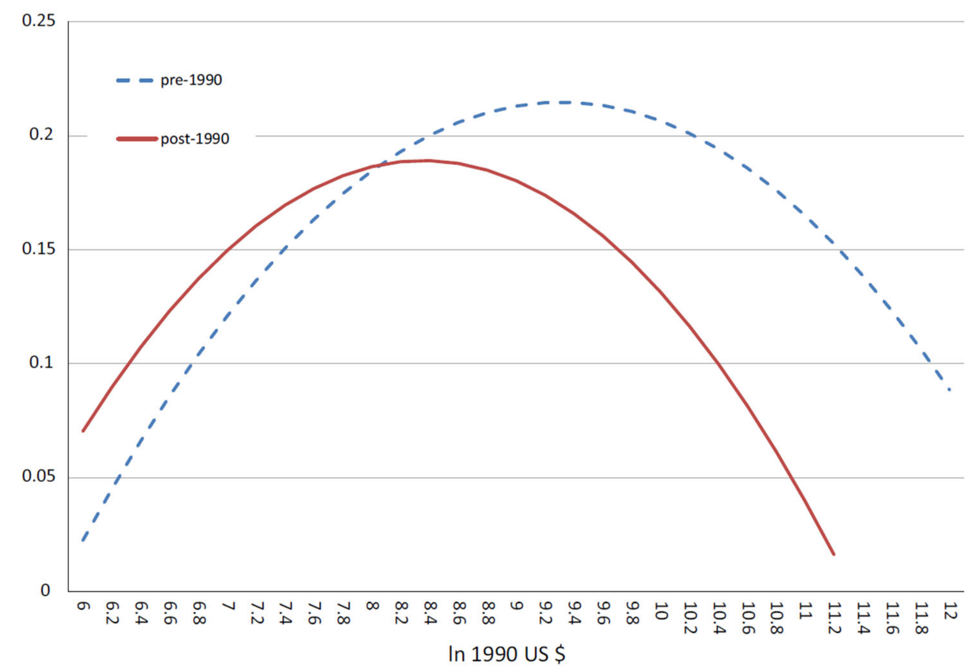


Fig. 6 Simulated manufacturing employment shares

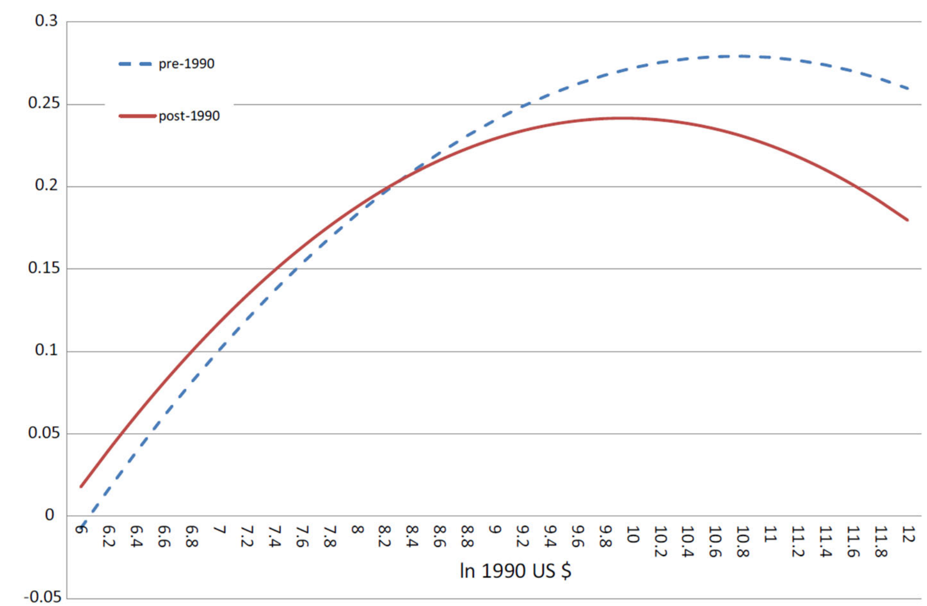
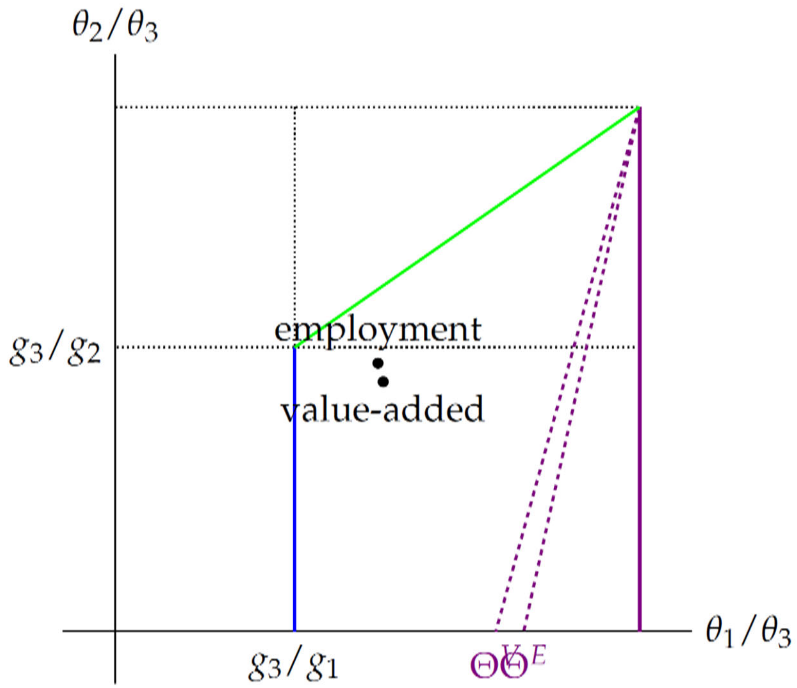
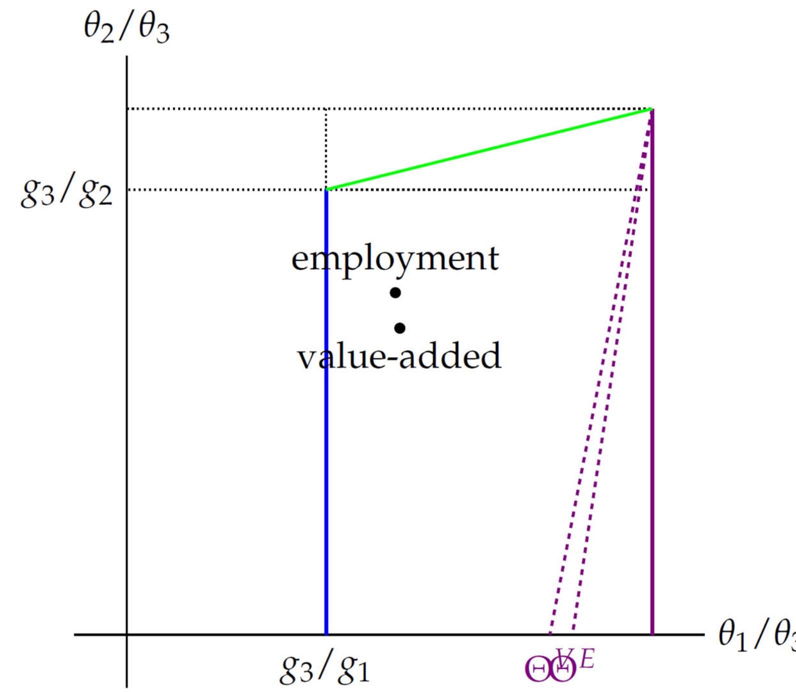


Fig. 7 Simulated manufacturing output shares (MVA/GDP at constant prices)

	Duarte-Restuccia (2010): $g_1 = 3.8\% > g_2 = 2.4\% > g_3 = 1.3\%$	**Comin et. al. (2021)** $g_1 = 2.9\% > g_2 = 1.3\% > g_3 = 1.1\%$
Empl. Shares	$\left(e^{-g_1\theta_1\lambda}, e^{-g_2\theta_2\lambda}, e^{-g_3\theta_3\lambda}\right) \approx (13.9\%, 28.1\%, 26.0\%)$; $(\theta_1/\theta_3, \theta_2/\theta_3) \approx (0.501, 0.512)$; $\Theta \approx 0.779$.	$\left(e^{-g_1\theta_1\lambda}, e^{-g_2\theta_2\lambda}, e^{-g_3\theta_3\lambda}\right) \approx (17.5\%, 36.9\%, 27.4\%)$ $(\theta_1/\theta_3, \theta_2/\theta_3) \approx (0.511, 0.650)$ and $\Theta \approx 0.848$.
VA Shares	$\left(e^{-g_1\theta_1\lambda}, e^{-g_2\theta_2\lambda}, e^{-g_3\theta_3\lambda}\right) \approx (15.1\%, 32.9\%, 28.2\%)$; $(\theta_1/\theta_3, \theta_2/\theta_3) \approx (0.511, 0.476)$ and $\Theta \approx 0.726$.	$\left(e^{-g_1\theta_1\lambda}, e^{-g_2\theta_2\lambda}, e^{-g_3\theta_3\lambda}\right) \approx (18.9\%, 43.3\%, 29.6\%)$; $(\theta_1/\theta_3, \theta_2/\theta_3) \approx (0.520, 0.583)$ and $\Theta \approx 0.805$





$\theta_1 g_1 > \theta_3 g_3 > \theta_2 g_2 \iff e^{-\theta_1 g_1 \lambda} < e^{-\theta_3 g_3 \lambda} < e^{-\theta_2 g_2 \lambda}$.

Cross-country productivity differences not only the largest in A but also the smallest in M.

1st Extension: Introducing the Engel Effect

The Engel Law through Isoelastic Nonhomothetic CES; Comin-Lashkari-Mestieri (2021), Matsuyama (2019)

$$\left[\sum_{j=1}^{3}(\beta_j)^{\frac{1}{\sigma}}\left(\frac{c_j}{U^{\varepsilon_j}}\right)^{1-\frac{1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \equiv 1$$

Normalize $\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3$; with $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$, we go back to the standard homothetic CES.
With $\sigma < 1$, $0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3$ $\Rightarrow$ **the income elasticity the lowest in A and the highest in S.**

By maximizing U subject to $\sum_{j=1}^{3} p_j c_j \leq E$,

Expenditure Shares $$m_j \equiv \frac{p_j c_j}{E} = \frac{\beta_j\left(U^{\varepsilon_j}p_j\right)^{1-\sigma}}{\sum_{k=1}^{3}\beta_k\left(U^{\varepsilon_k}p_k\right)^{1-\sigma}} = \beta_j\left(\frac{U^{\varepsilon_j}p_j}{E}\right)^{1-\sigma} \Longrightarrow \frac{m_j}{m_k} = \frac{\beta_j}{\beta_k}\left(\frac{p_j}{p_k}U^{\varepsilon_j-\varepsilon_k}\right)^{1-\sigma}$$

Indirect Utility Function:

$$\left[\sum_{j=1}^{3}\beta_j\left(\frac{U^{\varepsilon_j}p_j}{E}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \equiv 1$$

Cost-of-Living Index:

$$\left[\sum_{j=1}^{3}\beta_j\left(\frac{U^{\varepsilon_j-1}p_j}{P}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \equiv 1 \Leftrightarrow U \equiv \frac{E}{P}$$

Income Elasticity:

$$\eta_j \equiv \frac{\partial \ln c_j}{\partial \ln(U)} = 1 + \frac{\partial \ln m_j}{\partial \ln(E/P)} = 1 + (1-\sigma)\left\{\varepsilon_j - \sum_{k=1}^{3} m_k\varepsilon_k\right\}$$

Structural Change with the Engel (Income) Effect: Let $0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 = 3 - \varepsilon_1 - \varepsilon_2$.
Then, *even with constant relative prices,*

Decline of Agriculture: $s_1(t) = m_1(t)$ is decreasing in $U(t)$, because

$$\frac{1}{s_1(t)} - 1 = \frac{m_2(t)}{m_1(t)} + \frac{m_3(t)}{m_1(t)} = \frac{\beta_2}{\beta_1}\left(\frac{p_2}{p_1}U(t)^{\varepsilon_2-\varepsilon_1}\right)^{1-\sigma} + \frac{\beta_3}{\beta_1}\left(\frac{p_3}{p_1}U(t)^{\varepsilon_3-\varepsilon_1}\right)^{1-\sigma}$$

Rise of Services: $s_3(t) = m_3(t)$ is increasing in $U(t)$, because

$$\frac{1}{s_3(t)} - 1 = \frac{m_1(t)}{m_3(t)} + \frac{m_2(t)}{m_3(t)} = \frac{\beta_1}{\beta_3}\left(\frac{p_1}{p_3}U(t)^{\varepsilon_1-\varepsilon_3}\right)^{1-\sigma} + \frac{\beta_2}{\beta_3}\left(\frac{p_2}{p_3}U(t)^{\varepsilon_2-\varepsilon_3}\right)^{1-\sigma}$$

Rise and Fall of Manufacturing: $s_2(t) = m_2(t)$ is hump-shaped in $U(t)$, because

$$\frac{1}{s_2(t)} - 1 = \frac{m_1(t)}{m_2(t)} + \frac{m_3(t)}{m_2(t)} = \frac{\beta_1}{\beta_2}\left(\frac{p_1}{p_2}U(t)^{\varepsilon_1-\varepsilon_2}\right)^{1-\sigma} + \frac{\beta_3}{\beta_2}\left(\frac{p_3}{p_2}U(t)^{\varepsilon_3-\varepsilon_2}\right)^{1-\sigma}.$$

Hump-shaped due to the two opposing forces: $\varepsilon_1 < \varepsilon_2$ pushes labor out of A to M; $\varepsilon_2 < \varepsilon_3$ pulls labor out of M to S.

$$s_2'(t) = m_2'(t) \gtreqless 0 \Leftrightarrow (\varepsilon_2 - \varepsilon_1)\frac{m_1(t)}{m_2(t)} \gtreqless (\varepsilon_3 - \varepsilon_2)\frac{m_3(t)}{m_2(t)} \Leftrightarrow \eta_2 \gtreqless 1$$

Initially, when A is large & S is small, the former effect is stronger. Over time, A shrinks & S expands, and eventually, the latter effect becomes stronger.

(Analytically Solvable) Case: $0 < \mu \equiv \frac{\varepsilon_2 - \varepsilon_1}{g_1 - g_2} = \frac{\varepsilon_3 - \varepsilon_2}{g_2 - g_3} < \frac{1}{g_1 - \bar{g}}$, where $\bar{g} \equiv \frac{g_1 + g_2 + g_3}{3}$

Proposition 3 (Impact of Adding the Engel Effect on top of the Baumol Effect)

Peak Time

$$\hat{t}(\lambda;\mu) = \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3}\lambda - \mu \ln \widehat{U}(\lambda;\mu) = \hat{t}(\lambda;0) - \frac{\mu}{1+\mu\bar{g}} \ln \widehat{U}(\lambda;0)$$

Peak M-Share

$$\frac{1}{\widehat{s_2}(\lambda;\mu)} = 1 + \left(\frac{\tilde{\beta}_1}{\tilde{\beta}_2}\right) e^{a(g_1-g_2)\left(\frac{\theta_1 g_1 - \theta_2 g_2}{g_1 - g_2}\lambda - \hat{t}(\lambda;0)\right)} + \left(\frac{\tilde{\beta}_3}{\tilde{\beta}_2}\right) e^{a(g_2-g_3)\left(\hat{t}(\lambda;0) - \frac{\theta_2 g_2 - \theta_3 g_3}{g_2 - g_3}\lambda\right)} = \frac{1}{\widehat{s_2}(\lambda;0)}$$

Peak Time Per Capita Income

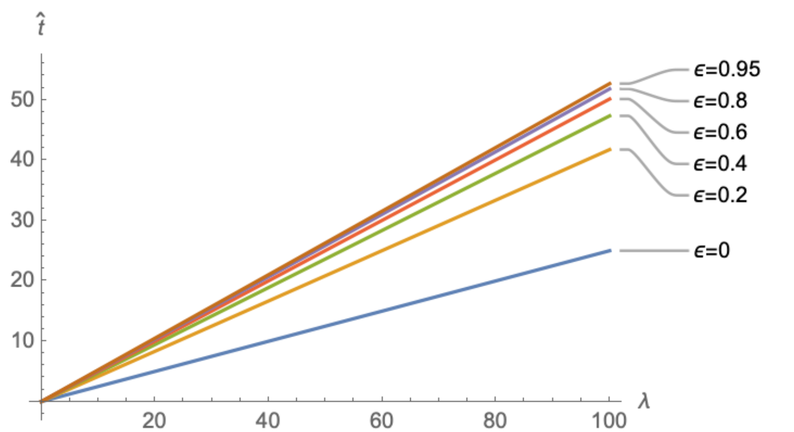
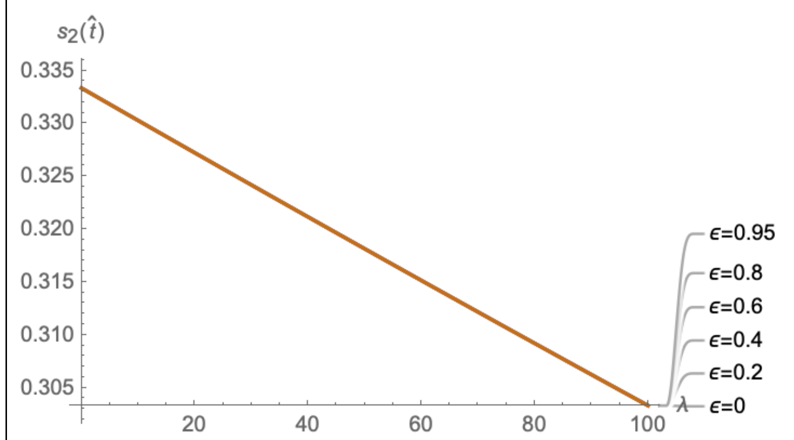
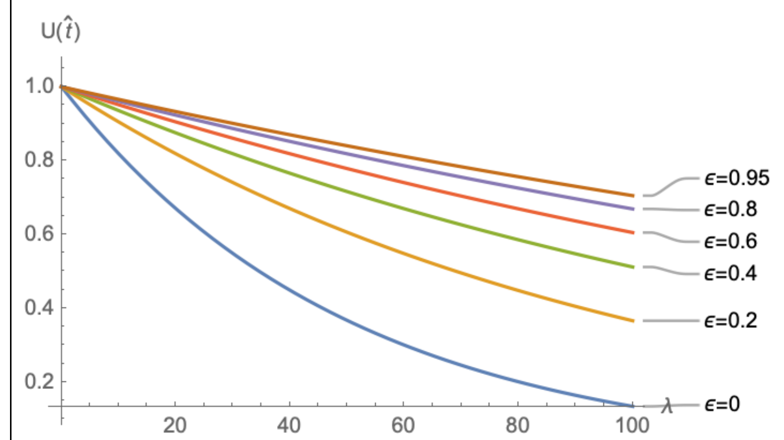
$$\widehat{U}(\lambda;\mu) = \left\{\sum_{k=1}^{3} \tilde{\beta}_k e^{-a g_k[\hat{t}(\lambda;0) - \theta_k \lambda]}\right\}^{-\frac{1}{a}\left(\frac{1}{1+\mu\bar{g}}\right)} = \widehat{U}(\lambda;0)^{\left(\frac{1}{1+\mu\bar{g}}\right)}$$

- $\widehat{s_2}'(\lambda;\mu) < 0$; $\widehat{U}'(\lambda;\mu) < 0$ under the same condition; $\hat{t}'(\lambda;\mu) > 0$ under a weaker condition.
- Fixing g_1, g_2, g_3, a higher μ has
  - **No effect** on the peak values of the frontier country, $\hat{t}(0;\mu), \widehat{s_2}(0;\mu), \widehat{U}(0;\mu)$.
  - **A further delay** in $\hat{t}(\lambda;\mu)$ for every country with $\lambda > 0$.
  - **No effect** on $\widehat{s_2}(\lambda;\mu)$ for every country with $\lambda > 0$.
  - **A smaller decline** in $\widehat{U}(\lambda;\mu)$ for each country with $\lambda > 0$.

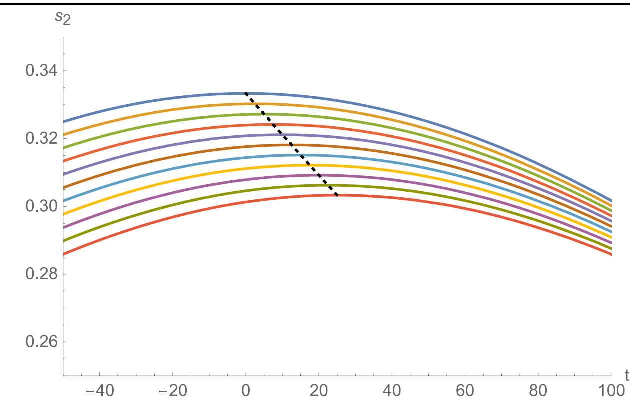
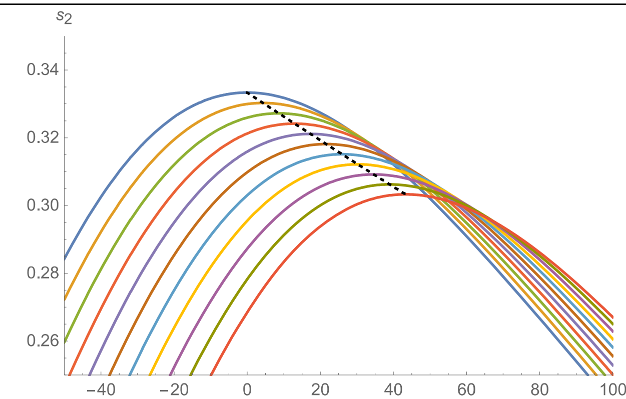
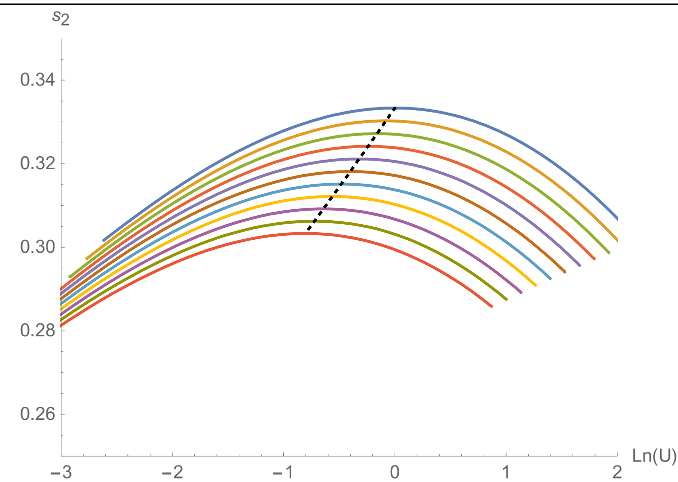
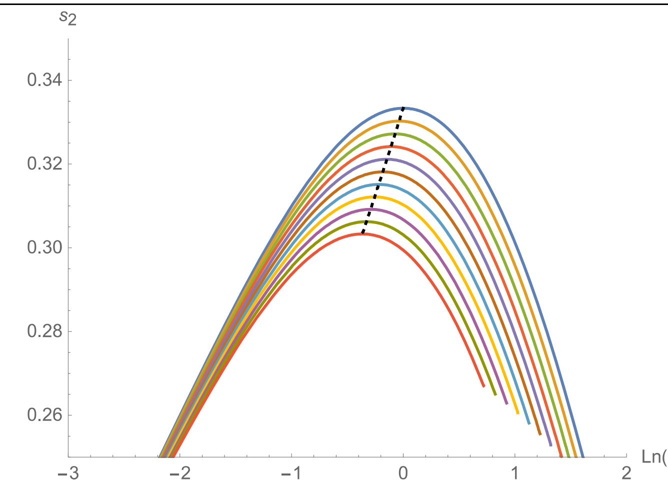
Analytically Solvable Case: A Numerical Illustration

$g_1 = 3.6\% > g_2 = 2.4\% > g_3 = 1.2\%$, $\theta = 0.5$, $a = 6/13$; $\tilde{\beta}_j = 1/3$ for $j = 1,2,3$.

In this case, $g_1 - g_2 = g_2 - g_3 = \bar{g} = 1.2\% > 0 \Longrightarrow \varepsilon_1 = 1 - \epsilon < \varepsilon_2 = 1 < \varepsilon_3 = 1 + \epsilon$ for $0 < \epsilon = (1.2\%)\mu < 1$

Peak Time	Peak M-Share	Peak Time Per Capita Income
		

Stronger nonhomotheticity changes the shape of the time paths significantly.
It does not change the implications on PD, i.e., how technology gaps affect $\hat{t}$, $s_2(\hat{t})$, and $U(\hat{t})$.

<table>
<tr><th></th><th>Homothetic ($\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$)</th><th>Nonhomothetic ($\varepsilon_1 = .4 < \varepsilon_2 = 1 < \varepsilon_3 = 1.6$)</th></tr>
<tr><td>$(t, s_2(t))$</td><td>
</td><td>
</td></tr>
<tr><td>$(\ln U(t), s_2(t))$</td><td>
</td><td>
</td></tr>
</table>

What happens if we had *solely* the Engel effect with $0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 = 3 - \varepsilon_1 - \varepsilon_2$, without the Baumol effect, $g_1 = g_2 = g_3 = \bar{g} > 0$?

Under the two normalizations

$$\left(\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_3 - \varepsilon_2}\right)\frac{\tilde{\beta}_1}{\tilde{\beta}_3} = 1; \;\; \tilde{\beta}_1 + \tilde{\beta}_2 + \tilde{\beta}_3 = 1$$

which ensures $\widehat{U}(0) = 1$ and $\hat{t}(0) = 0$,

Proposition 4: Peak Values under the Engel (Income) Effect only

Peak Time

$$\hat{t}(\lambda) = \frac{1}{a\bar{g}} \ln\left\{\sum_{k=1}^{3} \tilde{\beta}_k e^{a(\theta_k \bar{g}\lambda + \varepsilon_k \ln \widehat{U}(\lambda))}\right\}$$

Peak M-Share

$$\frac{1}{\widehat{s_2}(\lambda)} = 1 + \left(\frac{\tilde{\beta}_1}{\tilde{\beta}_2}\right) e^{a(\varepsilon_2-\varepsilon_1)\left(-\frac{\theta_2-\theta_1}{\varepsilon_2-\varepsilon_1}\bar{g}\lambda - \ln \widehat{U}(\lambda)\right)} + \left(\frac{\tilde{\beta}_3}{\tilde{\beta}_2}\right) e^{a(\varepsilon_3-\varepsilon_2)\left(\ln \widehat{U}(\lambda) - \frac{\theta_2-\theta_3}{\varepsilon_3-\varepsilon_2}\bar{g}\lambda\right)}$$

Peak Time Per Capita Income

$$\ln \widehat{U}(\lambda) = \frac{\theta_1 - \theta_3}{\varepsilon_3 - \varepsilon_1}\bar{g}\lambda$$

Proposition 5: Conditions for PD with the Engel (Income) Effect Only

$$\hat{U}'(\lambda) < 0 \text{ for all } \lambda > 0 \Leftrightarrow 0 < \frac{\theta_1}{\theta_3} < 1$$

With a low θ_1 and a high θ_3, the price of the income elastic S is high relative to the income inelastic A in a high-λ country, which make it necessary to reallocate labor to S at earlier stage of development.

$$\hat{s}_2'(\lambda) < 0 \text{ for all } \lambda > 0 \Leftrightarrow \left(1 - \frac{\varepsilon_2}{\varepsilon_3}\right)\left(\frac{\theta_1}{\theta_3} - 1\right) + \left(1 - \frac{\varepsilon_1}{\varepsilon_3}\right)\left(1 - \frac{\theta_2}{\theta_3}\right) > 0.$$

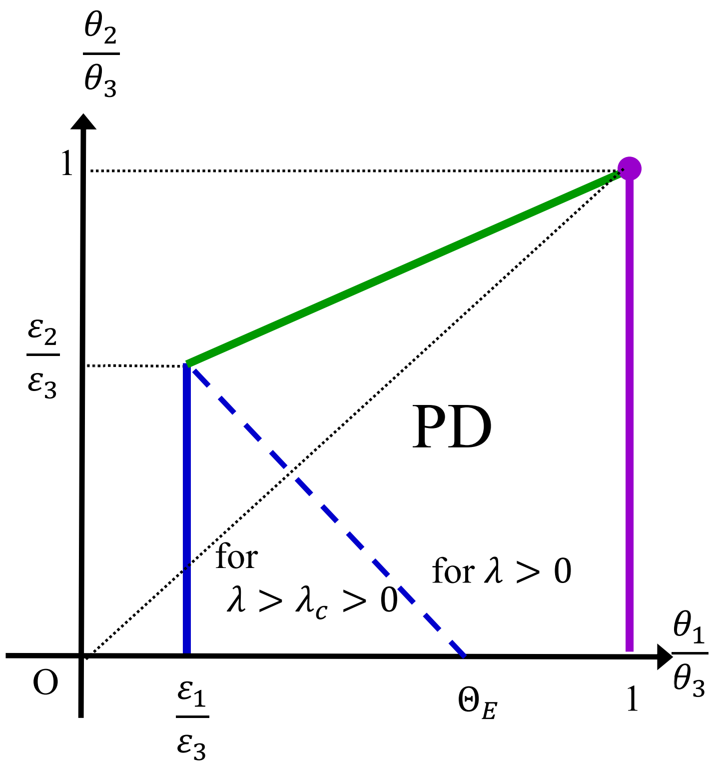
With a low θ_2, which has no effect on $\hat{U}(\lambda)$, the price of M is low relative to both A & S in a high-λ country, which keeps the M-share low.

Under the above condition,

$$\hat{t}'(\lambda) > 0 \text{ for a sufficiently large } \lambda \Leftrightarrow \frac{\theta_1}{\theta_3} > \frac{\varepsilon_1}{\varepsilon_3}$$

$$\hat{t}'(\lambda) > 0 \text{ for all } \lambda > 0 \Leftrightarrow \left(\Theta_E - \frac{\varepsilon_1}{\varepsilon_3}\right)\left[1 - \left(\frac{\varepsilon_3}{\varepsilon_2}\right)\frac{\theta_2}{\theta_3}\right] < \frac{\theta_1}{\theta_3} - \frac{\varepsilon_1}{\varepsilon_3} < 1 - \frac{\varepsilon_1}{\varepsilon_3}$$

where $\varepsilon_1/\varepsilon_3 < \Theta_E < 1$.



With $g_1 = g_2 = g_3 = \bar{g}$, PD occurs only if $\theta_1\bar{g}, \theta_2\bar{g} < \theta_3\bar{g}$, that is, when cross-country productivity difference is *the largest in S.*

2nd Extension: Introducing International Trade

One Implication of PD (consistent with the empirical evidence):

$$\frac{\partial}{\partial \lambda} \ln\left(\frac{A_1(t)}{A_2(t)}\right) = -(\theta_1 g_1 - \theta_2 g_2) < 0.$$

- A low-λ country has comparative advantage in A and a high-λ country has comparative advantage in M.
- Opening up trade in A and in M would weaken PD by allowing high-λ country to export M.
- Consistent with the findings that East Asia "suffers" less from PD.

A Two-Country Technology Gap Model of PD: $\lambda^1 < \lambda^2$ (Superscript indicates the country)

Trade Cost: Only $e^{-\tau_1} < 1$ fraction of A and only $e^{-\tau_2} < 1$ fraction of M shipped arrive to the destination.

$$m_j^c = \beta_j \left(\frac{p_j^c}{P^c}\right)^{1-\sigma} ; \quad P^c = \left[\sum_{k=1}^{3} \beta_k (p_k^c)^{1-\sigma}\right]^{1/(1-\sigma)} \quad \& \quad s_j^c = \left(A_j^c\right)^{\frac{1}{\alpha}} \left(\frac{p_j^c}{E^c}\right)^{\frac{1}{\alpha}}; \ E^c = \left[\sum_{k=1}^{3} (A_k^c)^{\frac{1}{\alpha}} (p_k^c)^{\frac{1}{\alpha}}\right]^{\alpha}$$

With $g_1\theta_1 > g_2\theta_2$, Leader (Country-1) has CA in A and Laggard (Country-2) has CA in M.

1 may export A to 2: $e^{\tau_1} p_1^1 \geq p_1^2;\ e^{-\tau_1}[A_1^1(s_1^1)^{1-\alpha} - c_1^1]L^1 = [c_1^2 - A_1^2(s_1^2)^{1-\alpha}]L^2 \geq 0.$ →# $[s_1^1 - m_1^1]E^1L^1 = [m_1^2 - s_1^2]E^2L^2 \geq 0.$#

2 may export M to 1: $p_2^1 \leq e^{\tau_2} p_2^2; [c_2^1 - A_2^1(s_2^1)^{1-\alpha}]L^1 = e^{-\tau_2}[A_2^2(s_2^2)^{1-\alpha} - c_2^2]L^2 \geq 0.$ → $[m_2^1 - s_2^1]E^1L^1 = [s_2^2 - m_2^2]E^2L^2 \geq 0.$

S is nontradeable: $p_3^1 \neq p_3^2;\ c_3^1 = A_3^1(s_3^1)^{1-\alpha};\ c_3^2 = A_3^2(s_3^2)^{1-\alpha}$ → $m_3^1 = s_3^1;\ m_3^2 = s_3^2.$

Condition for No Trade Equilibrium:

$$e^{\tau_1+\tau_2} > \frac{p_2^1(t)}{p_1^1(t)}\frac{p_1^2(t)}{p_2^2(t)} = \left[\frac{A_2^1(t)}{A_1^1(t)}\frac{A_1^2(t)}{A_2^2(t)}\right]^{-\frac{a}{(1-\sigma)}} = e^{\frac{a(g_1\theta_1-g_2\theta_2)}{(1-\sigma)}(\lambda^2-\lambda^1)}$$

$$\Leftrightarrow \tau_1+\tau_2 > T_+ \equiv \frac{a(g_1\theta_1-g_2\theta_2)}{(1-\sigma)}(\lambda^2-\lambda^1) > 0.$$

Trade Equilibrium under

$$0 < \tau_1+\tau_2 \leq T_+ \equiv \frac{a(g_1\theta_1-g_2\theta_2)}{(1-\sigma)}(\lambda^2-\lambda^1).$$

Then, 1 exports A to 2 and imports M from 2.

Equilibrium Conditions:

$$s_1^1+s_2^1 = m_1^1+m_2^1;\; s_1^2+s_2^2 = m_1^2+m_2^2$$

$$[s_1^1-m_1^1]E^1L^1 = [s_2^2-m_2^2]E^2L^2 > 0$$

$$e^{\tau_1}p_1^1 = p_1^2;\; p_2^1 = e^{\tau_2}p_2^2$$

Impact of International Trade (Numerical Simulation): $L^1/L^2 = 1$.

$$0 < \tau \equiv \frac{\tau_1 + \tau_2}{T_+} \equiv \frac{(1-\sigma)(\tau_1 + \tau_2)}{a(g_1\theta_1 - g_2\theta_2)(\lambda^2 - \lambda^1)} < 1$$

$$\Longrightarrow\ 1 < \frac{p_2^1}{p_1^1}\frac{p_1^2}{p_2^2} = e^{\tau_1+\tau_2} = e^{\tau T_+} \leq e^{T_+}.$$

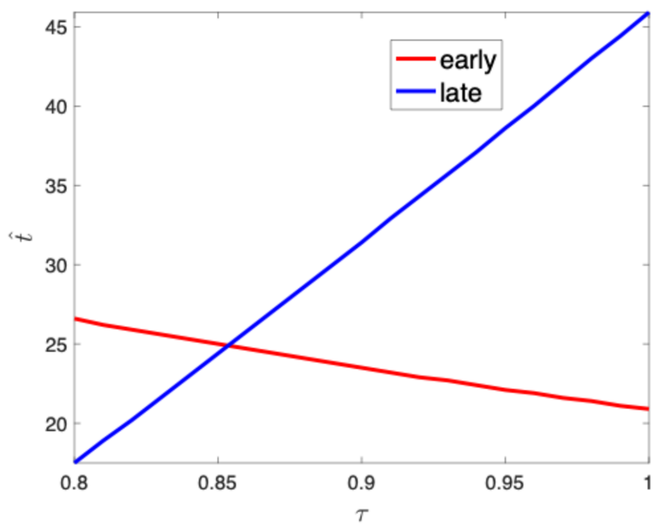
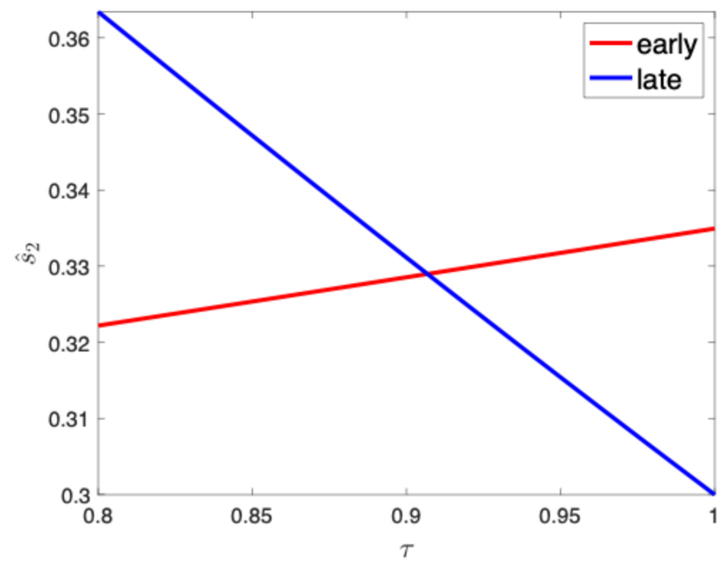
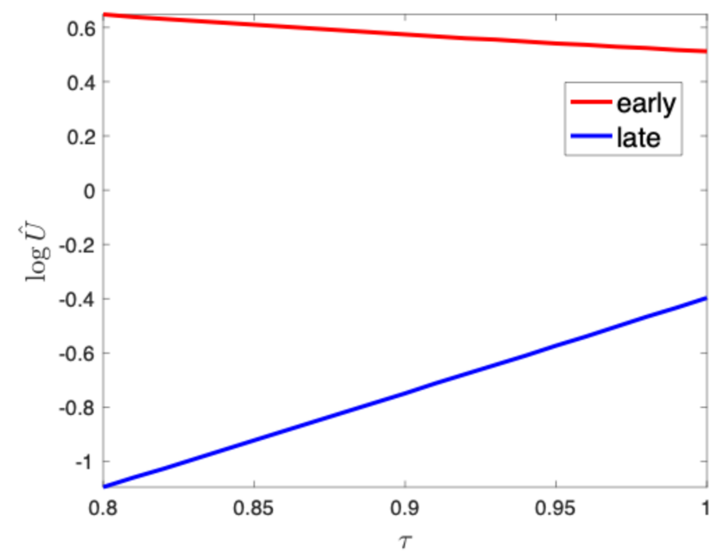
We plot how the peak values change in response to τ.

In all four cases, our mechanism for PD is:
- Robust to introducing international trade.
- Weaker in that the differences btw the leader and the laggard in $\hat{t}$ and $\hat{s}_2$ become smaller (larger in $\widehat{U}$ in $\widehat{m}_2$), as τ declines. For a sufficiently small τ, the reversal occurs in $\hat{t}$ and $\hat{s}_2$.

PD holds, when the trade cost accounts for more than about 1/3 of the imported goods prices, empirically plausible.

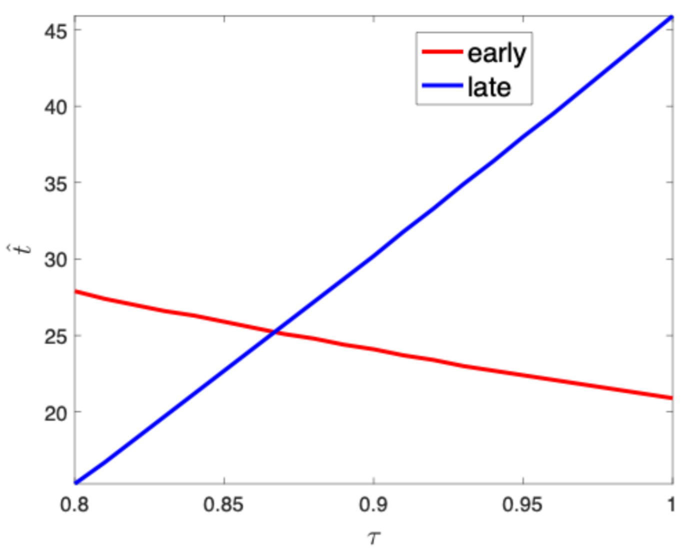
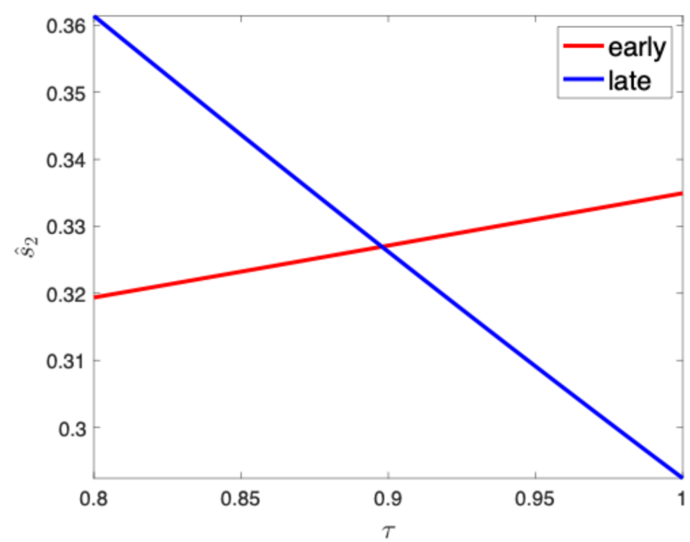
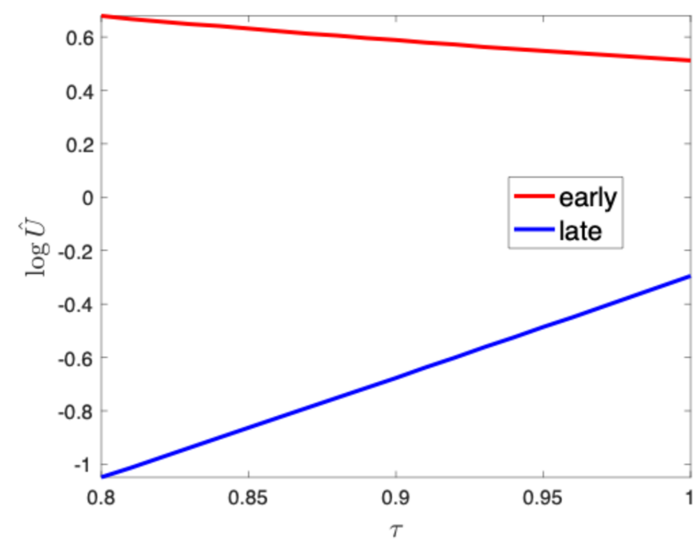
Under our mechanism, PD occurs not because of international trad but in spite of international trade.

Duarte-Restuccia productivity growth rates;
Employment Shares

Peak Time	Peak M-Share	Peak Time Per Capita Income
		

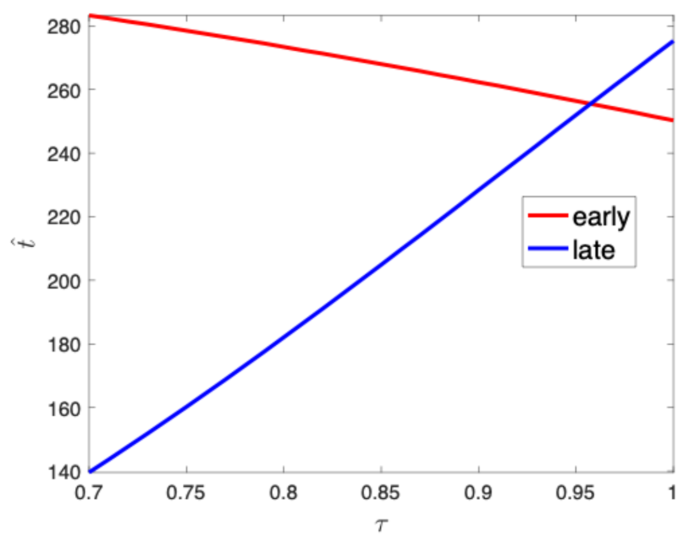
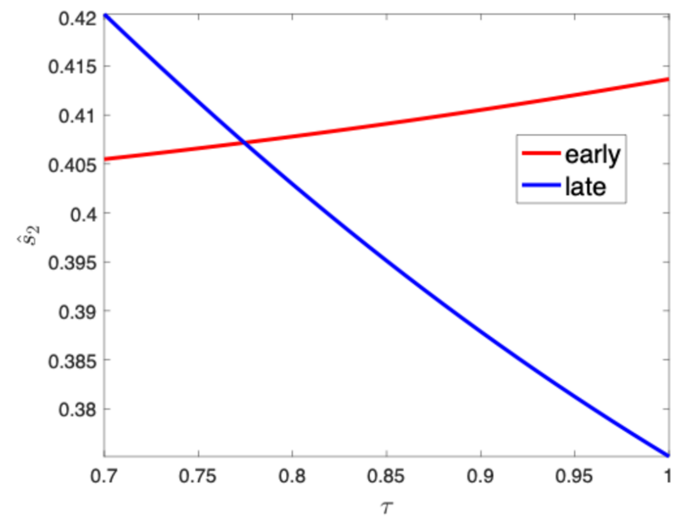
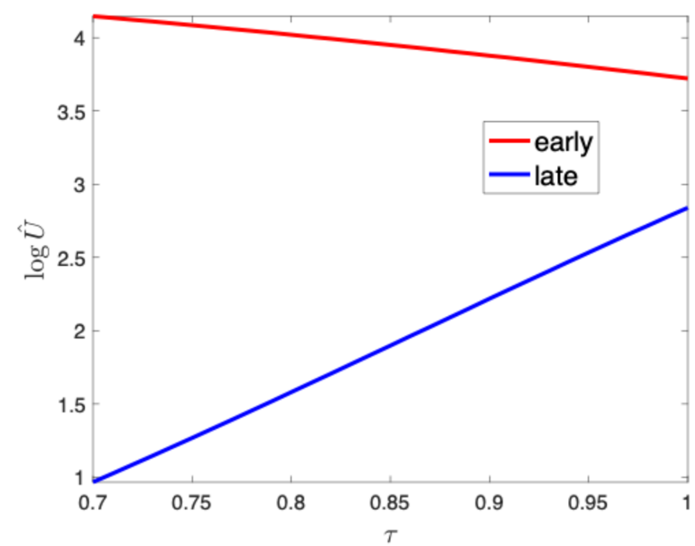
Reversal of $\hat{t}$ at $\tau \approx 0.85$, or $e^{\tau_1+\tau_2} = e^{\tau T_+} \approx 1.986 \rightarrow \sqrt{1.986} \approx 1.41$ times higher in the importing country.
Reversal of $\hat{s}_2$ at $\tau \approx 0.91$ or $e^{\tau_1+\tau_2} = e^{\tau T_+} \approx 2.242 \rightarrow \sqrt{2.242} \approx 1.497$ times higher in the importing country.

Duarte-Restuccia productivity growth rates;
Value-Added Shares

Peak Time	Peak M-Share	Peak Time Per Capita Income
		

Reversal of $\hat{t}$ at $\tau \approx 0.87$ or $e^{\tau_1+\tau_2} \approx 2.185 \rightarrow \sqrt{2.185} \approx 1.478$ times higher in the importing country.
Reversal of $\hat{s}_2$ at $\tau \approx 0.90$ or $e^{\tau_1+\tau_2} \approx 2.244 \rightarrow \sqrt{2.244} \approx 1.498$ times higher in the importing country.

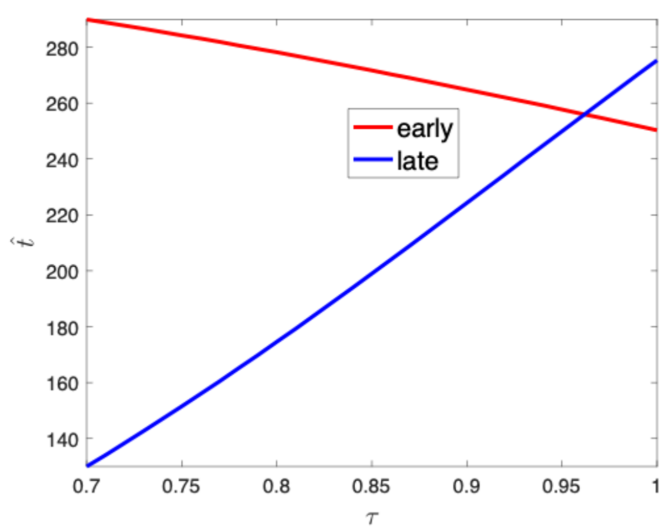
Comin-Lashkari-Mestieri productivity growth rates;
Employment Shares

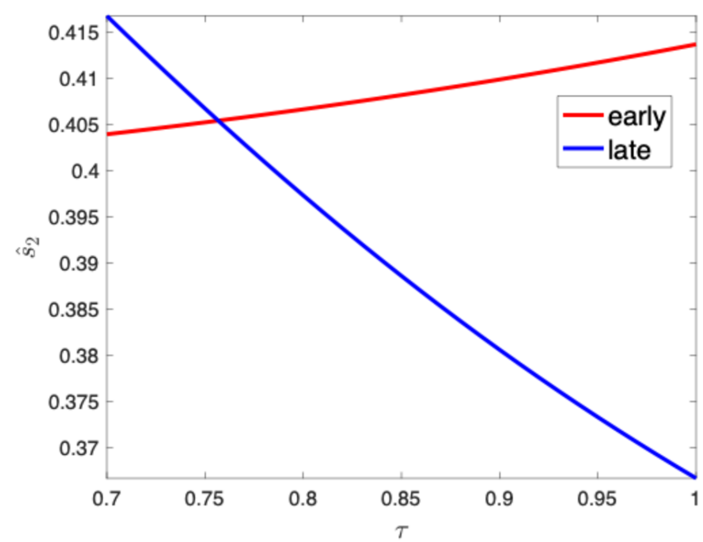
Peak Time	**Peak M-Share**	**Peak Time Per Capita Income**
		

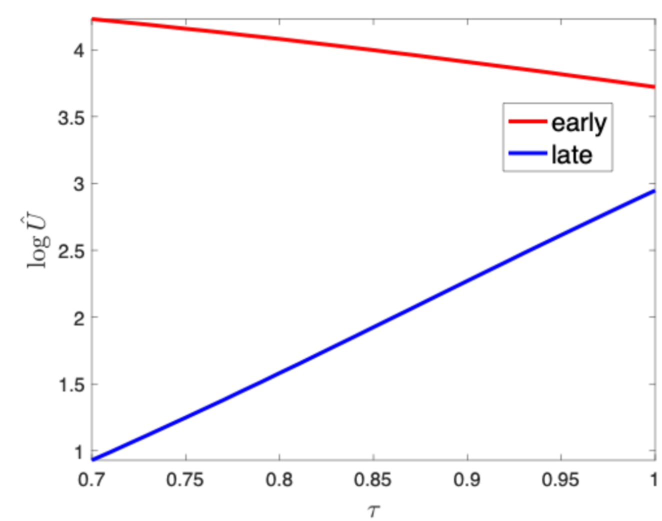
Reversal of $\hat{t}$ at $\tau \approx 0.96$, or $e^{\tau_1+\tau_2} \approx 2.295 \rightarrow \sqrt{2.295} \approx 1.515$ times higher in the importing country.
Reversal of $\hat{s}_2$ at $\tau \approx 0.77$ or $e^{\tau_1+\tau_2} \approx 1.947 \rightarrow \sqrt{1.947} \approx 1.395$ times higher in the importing country.

Comin-Lashkari-Mestieri productivity growth rates;
Value-Added Shares

Peak Time	**Peak M-Share**	**Peak Time Per Capita Income**







Reversal of $\hat{t}$ at $\tau \approx 0.96$, or $e^{\tau_1+\tau_2} \approx 2.504 \rightarrow \sqrt{2.504} \approx 1.582$ times higher in the importing country.
Reversal of $\hat{s}_2$ at $\tau \approx 0.76$ or $e^{\tau_1+\tau_2} \approx 2.068 \rightarrow \sqrt{2.068} \approx 1.438$ times higher in the importing country.

3rd Extension: Introducing Catching Up

Narrowing a Technology Gap

We assumed that λ is time-invariant.

This implies the productivity growth rate in each sector is constant over time & identical across countries.
[The aggregate growth rate, $g_U(t) \equiv U'(t)/U(t) = \sum_{k=1}^{3} g_k s_k(t)$, declines over time, $g'_U(t) = g_1 s'_1(t) + g_2 s'_2(t) + g_3 s'_3(t) = (g_1 - g_2)s'_1(t) + (g_3 - g_2)s'_3(t) < 0$, due to the so-called Baumol's cost disease.]

What if technological laggards can ***narrow a technology gap****, and hence achieve a higher productivity growth in each sector?*

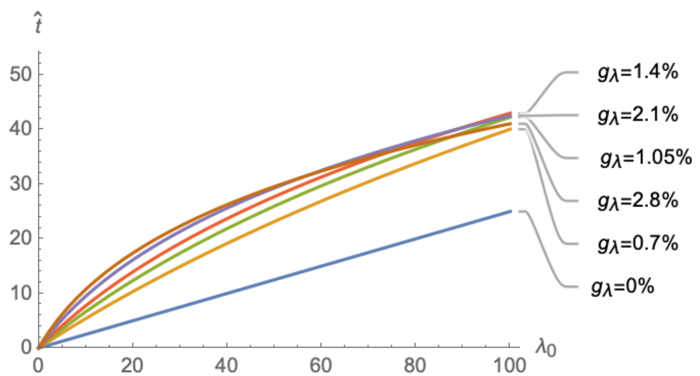
Countries differ only in the *initial* value of lambda, $\boldsymbol{\lambda_0}$, converging exponentially over time at **the same rate,**

$$A_j(t) = \bar{A}_j(0)e^{g_j(t-\theta_j\lambda_t)}, \quad \text{where } \lambda_t = \lambda_0 e^{-g_\lambda t}, \quad g_\lambda > 0.$$

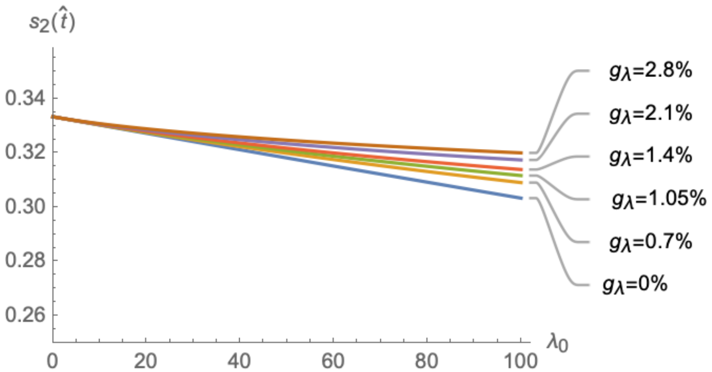
$$\Rightarrow \frac{1}{s_2(t)} = \left(\frac{\tilde{\beta}_1}{\tilde{\beta}_2}\right) e^{\alpha[(\theta_1 g_1 - \theta_2 g_2)\lambda_t - (g_1 - g_2)t]} + 1 + \left(\frac{\tilde{\beta}_3}{\tilde{\beta}_2}\right) e^{\alpha[(\theta_3 g_3 - \theta_2 g_2)\lambda_t + (g_2 - g_3)t]}$$

With double exponentials, we are able to solve the peak values only numerically.

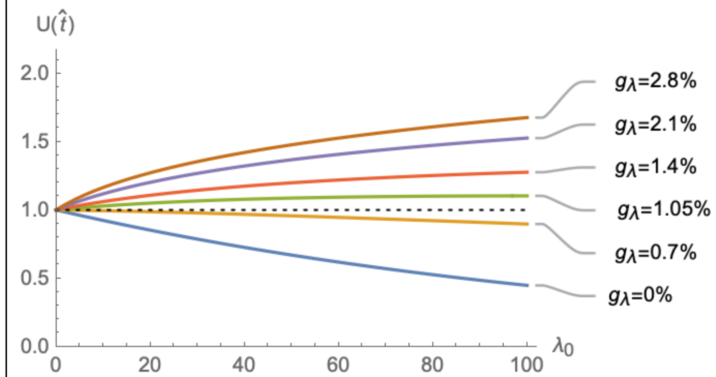
Peak Time



Peak M-Share



Peak time Per Capita Income



Technological laggards

- peak later in time,
- have lower peak M-shares
- have lower peak time per capita income, unless g_λ is too large: Comin-Mestieri (2018)

Concluding Remarks

A parsimonious mechanism for Rodrik’s (2016) PD based on

- **Differential productivity growth rates across complementary sectors**, as in Baumol (67), Ngai-Pissarides (07).
- **Countries heterogeneous only in their technology gaps**, as in Krugman (1985).
- Sectors differ in the extent to which technology gap affects their adoption lags, unlike in Krugman (1985)

In the baseline model, PD occurs for

- cross-country productivity difference larger in A than in S, which explains why a high-λ country peaks later in time.
- technology adoption takes not too long in M, which explains why a high-λ country has a low peak M-share.
- Technology adoption takes longer in S than in A, which explains why a high-λ country peaks prematurely.

The baseline model assumes **homothetic CES, no international trade, no catching up.**

In 3 extensions, we showed that the results are ***robust*** to introducing

- **The Engel effect** with income-elastic S & income-inelastic A, using nonhomothetic CES: CLM(21), Matsuyama(19)

The Engel effect changes the shape of the time paths, but not the implications on technology gaps on PD.
The Engel effect *alone* can’t generate PD w/o counterfactual implications on cross-country productivity differences.

- **International trade in A and in M**, but PD becomes weaker.
- **Narrowing a technology gap** to allow technological laggards to catch up,

unless the catching-up speed is too large.